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Investigation of undercritical microwave discharge ability to propagate limitlessly by continuous branching of the streamer*

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It is well known that at high gas pressure the microwave discharge is able to propagate in a field with amplitude less than critical one. Essential of this property is usually explained by a streamer mechanism analogically to spark discharge in DC field. But if in DC field the electric field on the top of a streamer is rising with increase of the streamer length without limit on its length, amplitude of microwave field on the top a thin conducting channel is increasing with length of a channel only while length is less than half wavelength of radiation, decreasing up to small values at bigger length. At this condition the question is arising: can the streamer create such configuration of channel, which helps to it having overcritical field on the top continuously, sustaining the streamer effect. Executed numerical modeling demonstrates this possibility leaving off all doubts about streamer nature of undercritical microwave discharges.

Nomenclature

E = effective amplitude of electric field of microwave radiation
E_{cr} = critical value of electric field
I = streamer current
c = light velocity
ω, λ = microwave radiation frequency and wave length
σ = plasma electrical conductivity
T = gas temperature
N = gas number density
C_p, C_v = gas thermal capacity at a constant pressure and constant volume
D = free electron diffusion coefficient
K_i, K_a = reaction rates of ionization and electron attachment
a = streamer radius
V = streamer radial velocity
f = degree of ionization
β = recombination coefficient
p = gas pressure
ρ = gas density
t = time
l = distance along vibrator and streamer trail
L_v = length of vibrator
k = $2\pi/\lambda$ - wave number
i = $(-1)^{1/2}$

I. Introduction

The nomenclature of different types of microwave (MW) discharges is very wide. For comparatively short pulse duration (of order about tens microseconds) it can be represented by overcritical diffusion and streamer

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discharges, undercritical streamer discharges and attached (deeply undercritical discharges)[1]. For application the undercritical streamer discharges are the most interesting because they are able to propagate on a long distance in the comparatively slight MW radiation, specific power of which is much less than critical value.

In difference to discharges in a MW radiation field, which is more than critical value, (so named overcritical discharges) the discharges in field, which is less than critical value (named undercritical discharges), can develop only in the streamer form by means of the field increasing on the streamer top. Their development starts from the top of initiator, which must create the prime zone of overcritical field near its top. On Fig.1 the calculated field distribution around passive electromagnetic vibrator is demonstrated. In given case electromagnetic vibrator is a metal cylinder with spherical ends. It is oriented along electric field of linear polarized MW radiation. One can see a strong increase of electric field amplitude comparatively origin field of radiation at the top of vibrator. Its length is near to half wavelength of radiation. It can play role of initiator because is able to increase origin field in many times.

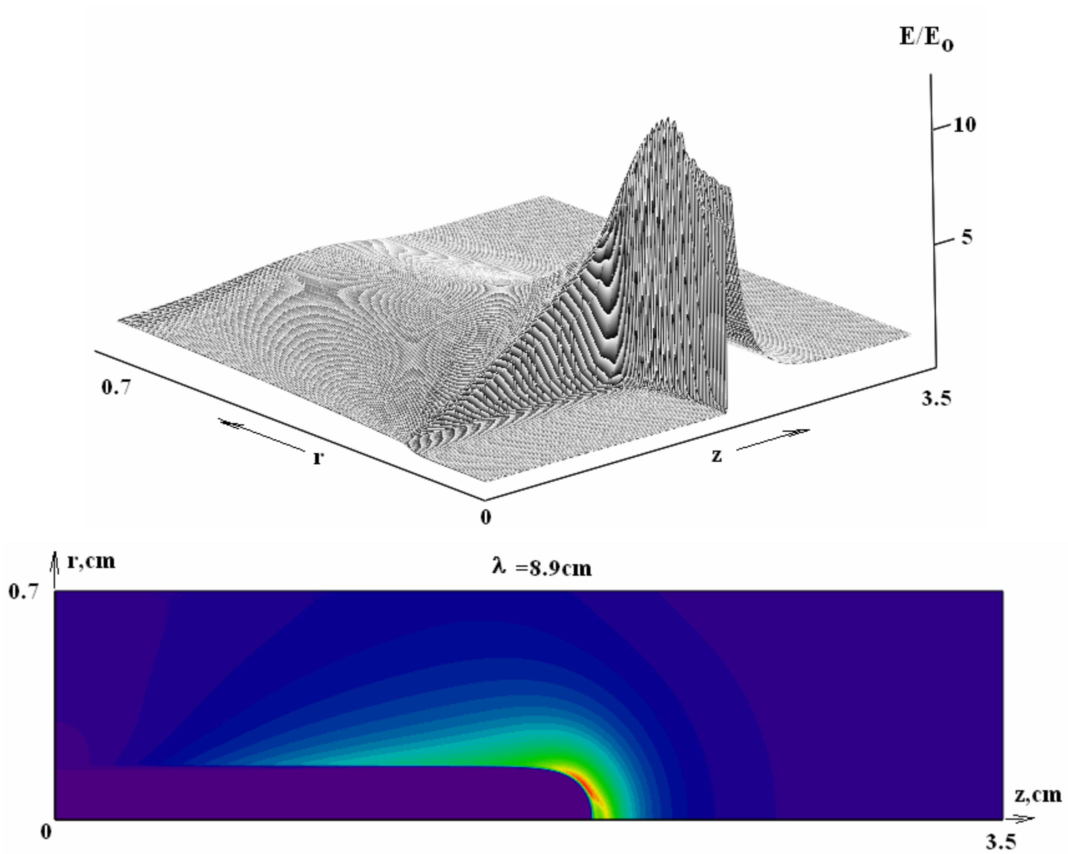


Figure 1. Electruc field amplitude distribution around cylindrical metal vibrator (serving as initiator) placed in MW radiation with wave length λ=8.9 cm The imaje corresponds to one half of vibrator. The origin electric fiel is oriented along axis Z.

Note that maximum field is outstanding from axis of vibrator. The value of outstanding depends on electrical length kL_v, ($k=2\pi/\lambda$) of vibrator's half-length. This dependence is shown on Fig.2. The outstanding is significant if electrical length of vibrator more unit. It means that this phenomenon is property of electromagnetic vibrators only and do not take place for DC field. As ionization of gas starts in a zone of field maximum the streamer must will its development in very that point of vibrator surface. The streamer propagation will develop in direction, which do not coaxial with vibrator. It is not clear in advance whether this property will play the role at head of streamer. Quite possibly, this property is able to induce the not straight propagation of streamer along trail of complicated trajectory.

The dependence of electric field increase on the top of metal cylinder oriented along origin electric field on its length for radiation with wave length λ=12.5 cm is compared with the same for case of DC field in Fig.3. Dependence of DC field increase for axial symmetric metal body of half-length $L>a_c$ is described by Eq.(1)

$$\frac{E(\varepsilon)}{E_0} = 1 - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{a_c}{L}}\right) - \frac{L}{a_c}\cdot\sqrt{1-\frac{a_c}{L}}}{\operatorname{arctanh}\left(\sqrt{1-\frac{a_c}{L}}\right) - \sqrt{1-\frac{a_c}{L}}}, \tag{1}$$

where a_c is curvature radius at the body's top.

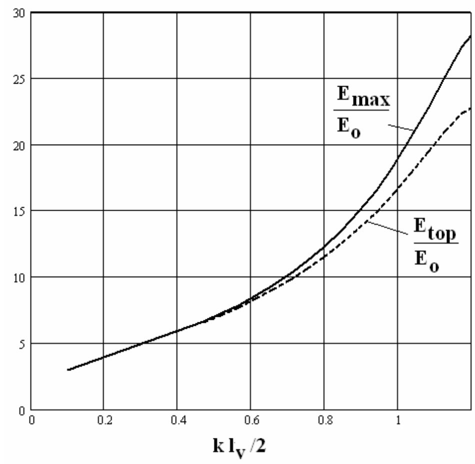


Figure 2. Electric field on the axis of and maximum field at ends of cylindrical passive vibrator via electrical length of it. Radius of vibrator *a*=0.1/*k*

For DC the more cylinder length, the more the field increase at cylinder top. This trend is independent on the streamer length (dashed line in Fig.3). Cardinally other situation we have for MW. At cylinder length less than value λ/2 the dependence is the same. But if the cylinder length is more than half-wavelength of radiation (electro-dynamic resonant value), the electric field amplitude on top of the cylinder is decreasing up to small value (external field amplitude and just smaller one). This circumstance creates rather different conditions for a MW streamer discharge development.

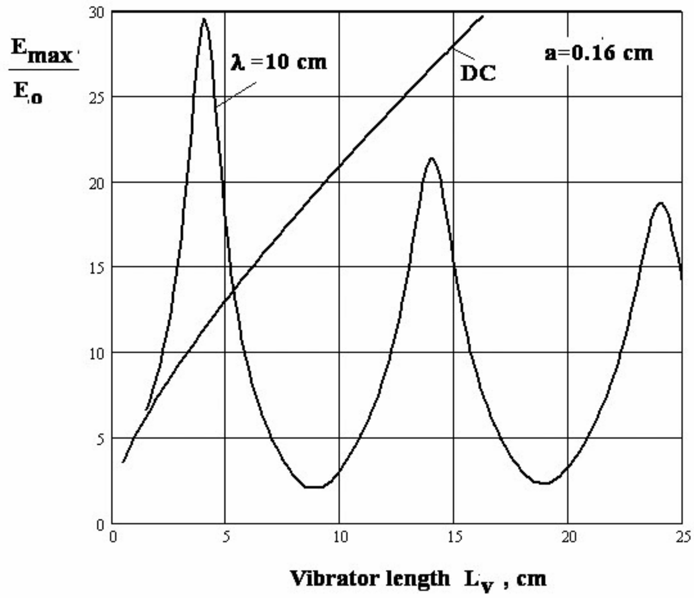


Figure 3. The field increase at ends of metal cylinder, placed along external electric field E_0 for DC and MW fields.

Without an initiation undercritical discharge can't start. But if the initiator creates the overcritical field at its top the streamer is starting from point of maximum field. It is seen on Fig.4. The streamer is starting indeed from the point on vibrators surface, where field amplitude is maximal, under angle to the vibrator axis accordingly to calculated distribution in Fig.1 and dependence on Fig.2.

Scenario of initial stage of a streamer development is described in Ref.[2], where main parameters of a streamer are estimated and it is simulated by numerical modeling[3]. Initial stage correlates to situation, when length of the streamer is less than half-wavelength of a microwave (MW) radiation. At that condition the dependence of the top field is the same as at DC and conditions for MW streamer development is almost the same as for spark streamer in DC field: the more streamer length, the is more electric field on its head, the is more speed of ionization. So streamer is rising continuously. DC streamer can rise unlimitedly along external electric field as alone conducting channel. But MW can't rise by such way because of top field decreasing if streamer length more resonant value.

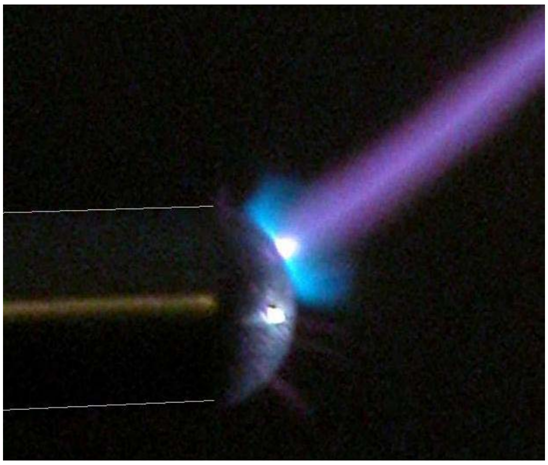


Figure 4. The streamer, which has started from the point of overcritical field on the surface of vibrators top. Parameters of vibrator are the same as on Fig.1

When the origin field is overcritical, this circumstance is not mention because the field is overcritical everywhere. But in case of the subcritical MW field a linear streamer must stop.

But experimentally we observe MW streamer discharge unlimitedly propagated far away from initiator on distance significantly more than half-wavelength of radiation. This effect demands explanation.

It was investigated earlier that initiated streamer discharges represent a complicated net of thin hot channels which fulfills some region. Bounds of the discharge region move against MW radiation with velocity about some km/s. The space density of the streamers in discharge region is high if original field is less than critical a little. But if the parameter undercriticality is near to low boundary of initiated discharge existing, the discharge is represented by several channels or just a single streamer. In last case the streamer forms the specific configuration: the sinusoidal channel with branches at each its extremum. First it was clearly observed by V.Brovkin and Yu. Kolesnichenko[4]. They pointed attention on self organization ability of the streamer forming the structure by analogy to the simplest wideband antenna system. It is seen very well on prepared by them photo of such discharge (middle on Fig.5, λ=4.3 cm). Later the same structures were observed in initiated discharges at other wavelength of radiation (left and right on Fig.5, λ=2.5 cm and 8.9 cm).

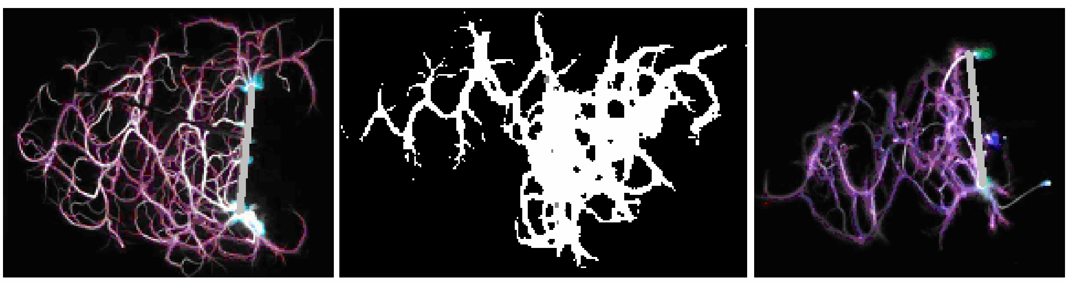


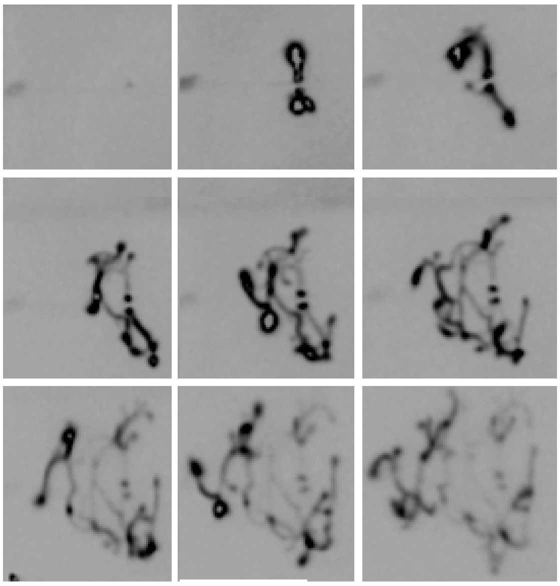
Figure 5. The typical view of undercritical MW discharge initiated by metal vibrator, oriented along electric field of external linear polarized electromagnetic wave. Radiation propagates from left to right, air pressure *p* = 200 Torr.

The obtained temporal development of the undercritical streamer discharge (see Fig.6) has shown the same picture: streamer channel forms the sinusoidal main channel with additional side branches; the brightness of the channels is maximal on the front of discharge area.

It was decided to check whether such form of the streamer (sinusoidal main channel with side branches, rising from points of extremum of main channel) enables to support continuous development of MW streamer.

II. Numerical model

The fully adequate model, which is able to describe the process of MW discharge development in undercritical field, demands use of the system of Helmholtz for electromagnetic field plus gas-dynamic system for multi-component plasma mix taking into account the needed physical-chemical processes in 3-D frame. Experience of designing of such model[5,6] have shown that difficulties of technology of calculation by such model do not allow investigating the process saving main its properties. Thus for realization of formulated task the simplified model of plasma thin channel dynamics was used.



p=160 Torr, $\tau_{exp} = \tau_{break} = 1\ \mu s$

Figure 6. Negative photo of undecritical MW discharge, initiated by metal sphere, one MW pulse in series of time moments. Exposition time – 1 μs, break time - 1 μs.

First it was described in Ref.[7] and more detail in Ref.[8]. Equation (2) describes processes of ionization by electron impact, electron diffusion, attachment and recombination in air

$$\frac{\partial f}{\partial t} = N \cdot \left(K_i\left(T_e\right) - K_a\right) \cdot f + \frac{\partial}{\partial l}\left(D\left(N, T_e\right) \cdot \frac{\partial f}{\partial l}\right) - \beta \cdot \left(f \cdot N\right)^2 \qquad (2)$$

Equation (3) takes into account the gas Ohm heating by inducted MW current

$$C_v\left(T_g, N\right)\frac{dT_g}{dt} = \frac{\sigma\left(f\right) \cdot \left|E\right|^2 \cdot \Phi\left(\sigma, a\right)}{N} - T_g \cdot \frac{2V}{a} \qquad (3)$$

Equations (4) – (8) correspond to so-called envelop model, often used for approximate simulation.

$$\frac{\partial N}{\partial t} = -2\frac{V}{a} \qquad (4)$$

$$\frac{dV}{dt} = \frac{2}{a} \cdot \left(\frac{P_0 - P}{\rho_0} - V^2\right) \qquad (5)$$

$$\frac{da}{dt} = V \tag{6}$$

$$p = N \cdot \left(T_g + f \cdot T_e\right) \tag{7}$$

$$T_e = F\left(\frac{E}{N}\right) \tag{8}$$

The system Eq.(2) - Eq.(8) is closed by 1-st kind integral equation (9) for current inducted in arbitrary oriented thin channels with given (calculated on each time step) distribution of conductivity

$$\begin{aligned} J(l) = \vartheta(l)\left(\vec{E}_0(\vec{r}(l)) \cdot \frac{d\vec{l}}{dl}\right) + \\ + i\frac{k}{c}\vartheta(l)\int J(l')\left[G(\vec{r}(l),\vec{r}(l'))\left(\frac{d\vec{l}\,'}{dl'} \cdot \frac{d\vec{l}}{dl}\right) + \frac{1}{k^2}\left(\frac{d\vec{l}}{dl} \cdot \nabla_r\left(\frac{d\vec{l}\,'}{dl'} \cdot \nabla_r(G(r, r(l')))\right)\right)\right]dl' \end{aligned} \tag{9}$$

where

$$G(\vec{r}, \vec{r}\,') = \frac{\exp(ikR(\vec{r}, \vec{r}\,'))}{R(\vec{r}, \vec{r}\,')},$$

$$R(\vec{r}, \vec{r}\,') = \sqrt{a^2 + |\vec{r} - \vec{r}\,'|^2},$$

$$\vartheta(l) = \pi \cdot a^2 \cdot \sigma(l)$$

Calculated current defines the field space distribution by known operators:

$$\vec{E}(\vec{r}) = \vec{E}_0(\vec{r}) + i \cdot \frac{k}{c} \cdot \int J(l') \cdot \left[G(\vec{r}, \vec{r}(l')) \cdot \frac{d\vec{l}\,'}{dl'} + \frac{1}{k^2} \cdot \nabla_r\left(\frac{d\vec{l}\,'}{dl'} \cdot \nabla_r(G(r, r(l')))\right)\right]dl' \quad . \tag{10}$$

Coordinates of streamer trails are defined parametrically

$$\vec{r} = \vec{r}(l)$$

Comparatively to Ref.[8] the used integral equation was modified for any complicated form of conducting thin channels. The integral equation defines the MW current distribution along channels with known distribution of conductivity.

Thus the model is based on main physical factors controlling the process of streamer development in undercritical field: ionization and electron diffusion, field increasing on the streamer tops, ionization and gas heating with caused gas dynamics.

The trails of possible developing streamers are elected the similar to observed and are shown on Fig.7.

Calculation was performed for air at initial pressure 200 Torr and surrounding gas temperature 300K and MW external field $E_o = 0.3E_{cr}$ at $\lambda = 8.9$cm. Period and swing of the sinusoidal trail and length of branches equal to $\lambda/8$ (see Fig.7). The length of initiator equals to $\lambda/4$. Its radius 0.15 cm is small enough for initiating of discharge at given origin field amplitude.

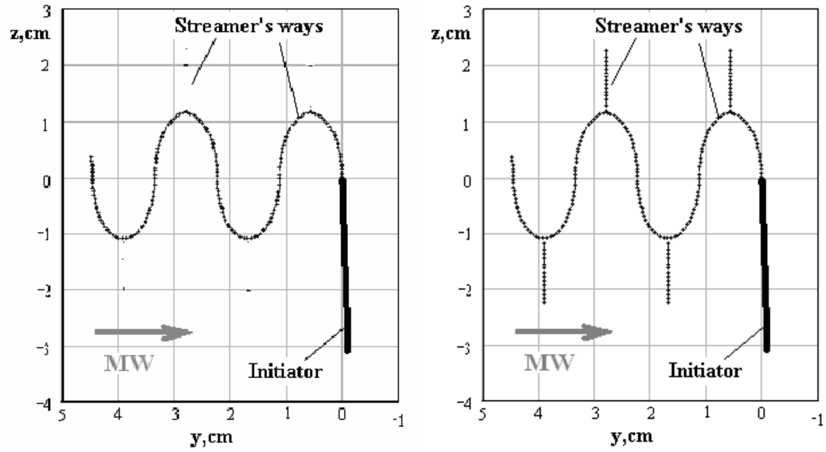


Fig.7. The fixed trails of the streamer without branches (left) and with branches (right).

III. Results of modeling

Modeling confirms the ability of the streamer discharge to propagate infinitely in undercritical field against MW radiation far away from initiator by means of branching.

On Fig.8 one can see the calculated distribution of MW current inducted in the branching streamer at varied moments of time (0, 13.4, 27.2, 41.0, 54.9 μs).

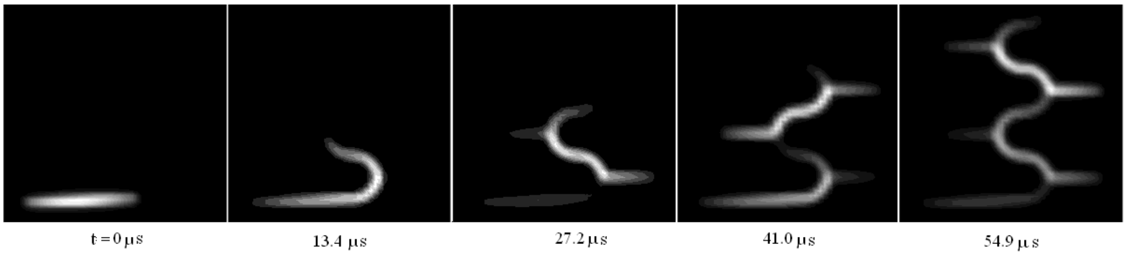


Figure 8. Calculated temporal development of the current distribution

Figure 9 demonstrates the spatial distribution of electric field amplitude (left) and calculated current (right) at modeling finish time moment (t = 54.9 μs). At t = 0 current exists only inside the initiator. But later the current (bright pats on the trail) propagates along the streamer way including branches away from initiator. The current is increasing in the next branch in that moment when electric field on top of the sinusoidal part is decreasing. Arise of branch is supporting the streamer system in electro-dynamical resonance.

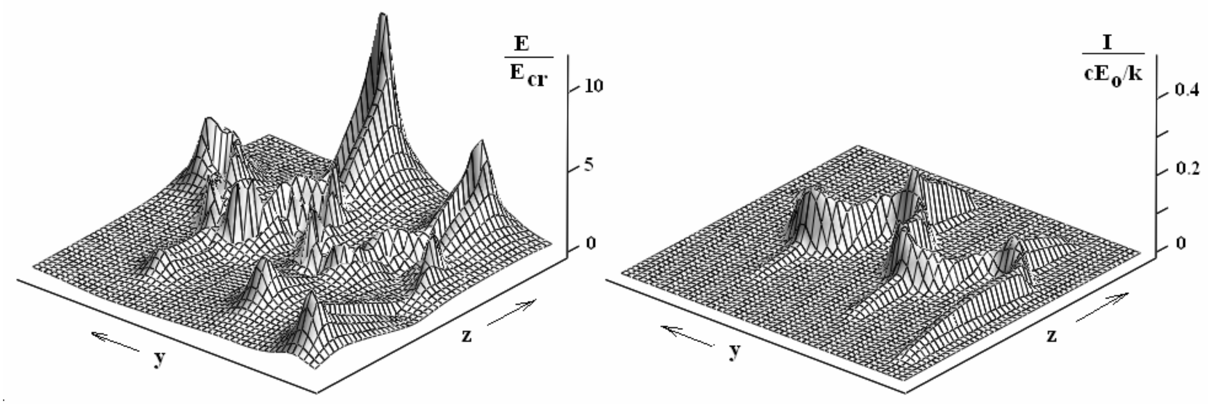


Figure 9. Space distributions of electric field (left) and current amplitude (right) of the streamer with branches at t=54.9 μs

As known from theory of wire antenna, the wire system, directed against radiation, with many transversal branches, length of which approximately equals to $\lambda/8 \div \lambda/4$, has resonant properties in a wide band of wave length. In such system only several first elements, located inside one wave length of radiation, are active. Other elements, been in shadow of first part, do not work. This process one can see on Fig.8. Maximum of current goes forward, shadowing the part, located nearer to initiator. The same is observed in experiments. The front of streamer net has thickness about half wavelength and shields the passed part of hot channels from radiation. Shielded part of discharge saves channels with high temperature almost without inducted current.

The average velocity of discharge front is 0.82 km/s and streamer velocity along the trail is 2.04 km/s. It well coincides with both observations and estimations in Ref.[2], based on taking into account of main factors: ionization frequency, electron free diffusion, Ohm heating and top field increasing.

For compare the calculated electric field and current distributions for streamer without branching at t=54.9 μs are shown in Fig.10. It is seen clearly that streamer propagation is stopped when its total length of vibrator and streamer exceeds resonant value~λ/2 (accordingly to Fig.3).

It is not clear what defines curvature of streamer trail. One can suppose that it coupled with the shift of maximum field from axis of a conducting channel as it is seen on Fig.1 and Fig.4 for metal body.

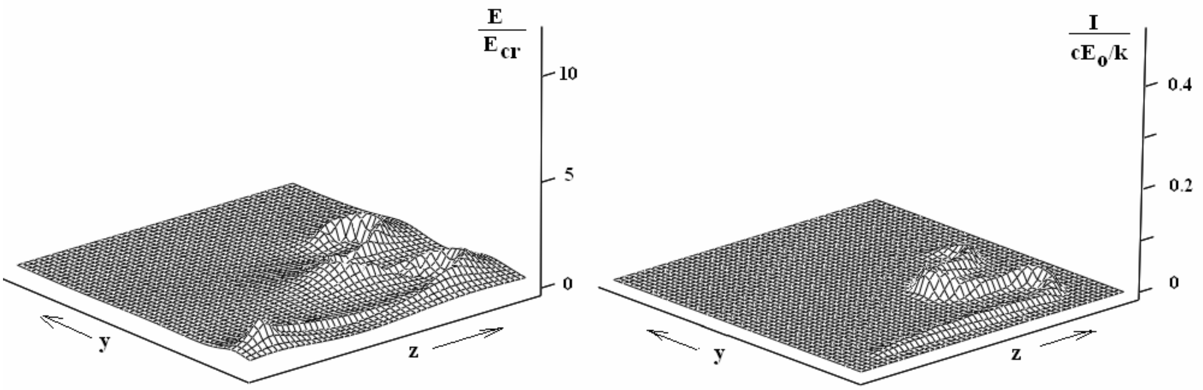


Figure 10. Space distributions of electric field (left) and current amplitude (right) of the streamer without branches at t=54.9 μs

Probably it is general property of electrodynamics of thin conducting channels with length compared with wavelength of radiation. As accordingly to Fig.3 this effect takes place if electrical length of channel is more than unit, it is naturally estimate the typical curvature radius equaled to $1/k$. Probably also this phenomena causes the instability of a streamer trajectory.

Observations show that really the curvature of the streamer trails depends on many factors, in particular, on parameter of undercriticality, gas pressure and others. This problem is a subject for farther investigations.

Figure 11-a and Fig.11-b demonstrate the typical distribution of main parameters of discharge at streamer's head along its trail. Important for estimations properties must be noted. Electric field is maximal at streamer's head and has a magnitude a bit more than critical value (see also Fig.11-c). At place of electric field maximum gas density is not decreased else and electrical conductivity equals to ω/4π. Behind the streamer's head parameter E/n, which defines the ionization balance, is constant and a bit more unite.

The head of streamer snake moves with average velocity equaled to 2 km/s (see Fig.11-d)

IV. Estimation of the undercritical MW streamer discharge parameters

Development of the streamer in undercritical field is limited by necessity of falling of density of gas in the streamer channel for maintenance of parameter E/N at a level of critical (see Fig.11-a and Fig.11-c). Without that the deionization will take place in the channel and it will lose ability to increase a field on the streamer's head up to values above critical. Speed of front of ionization on a streamer's head is described by Eq.(11)[2].

$$V_s = 2 \cdot \sqrt{D_e \cdot \nu_i} + \frac{|\mu| \cdot E}{\omega + \nu_i} \cdot \nu_i \,, \qquad (11)$$

where μ is electron mobility, ν_I – effective ionization frequency. The second term in Eq.(18) plays role only at high pressure, higher atmospheric one, at an electric field, normal to front of ionization (this situation is realized at the streamer head)[9].

But, the level of overcriticality in case of undercritical discharge, realized on a streamer's head, differs from unit a little, being established on a level at which speed of ionization front corresponds to rate of heating of the channel directly in a head part of streamer. The second term in Eq.(11) is negligibly small and in an estimation of growth velocity of undercritical streamer it will not be taken into account. The magnitude of ionization frequency on a head of streamer can be estimated on base of assumptions, which follow from modeling results, presented on Fig.11:

- heating, as well as ionization, occurs in a streamer's head in a field, poorly distinguished from critical value E_{cr},
- inside the streamer's head the conductivity rises up to value $\omega/4\pi$ by exponential law with index $\nu_i t$,
- the density behind head in the same limits should fall up to value E_o/E_{cr} of initial,
- gas heating is isobaric.

Basing on named assumptions, one can simplify the equation for temperature on axis of the streamer in a head part to form Eq.(12)

$$\frac{1}{T}\cdot\frac{dT}{dt}=\frac{\sigma\cdot E_{cr}{}^{2}}{C_p\cdot p} \qquad (12)$$

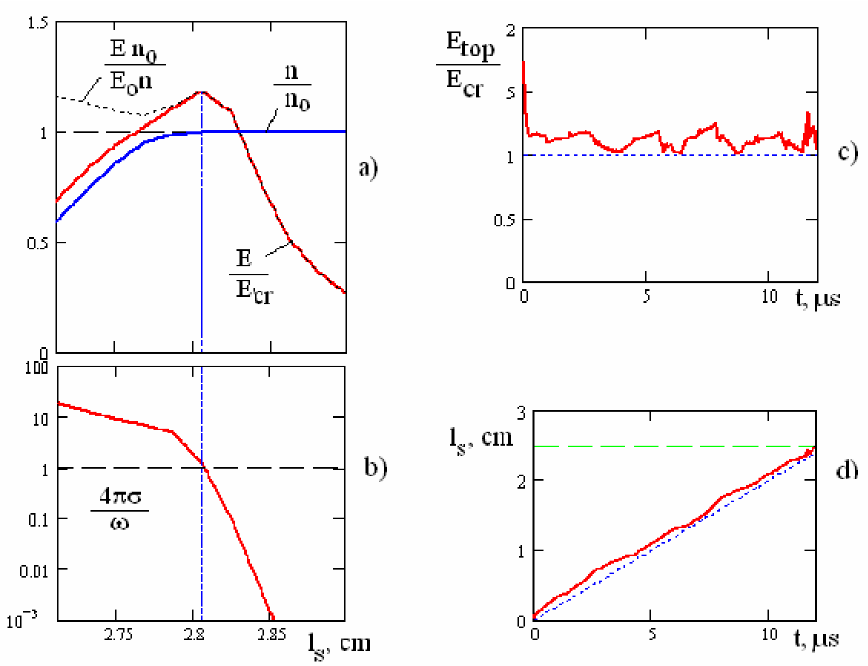


Figure 11. Discharge parameters: a) and b) – spatial distributions along the streamer's main trail near streamer's head, c) maximal electric field at streamer head in time, d) – streamer length in time

Integration Eq.(12) results in a ratio

$$\ln\frac{T}{T_0}=\ln\left(\frac{E_{cr}}{E}\right)=\frac{E_{cr}{}^{2}}{C_p\cdot p}\cdot\int_0^t\sigma_0\exp(\nu_i\cdot t)dt=\frac{E_{cr}{}^{2}}{C_p\cdot p}\cdot\frac{\omega}{4\cdot\pi\cdot\nu_i} \qquad (13)$$

From here we deduce the estimation for frequency of ionization

$$\nu_i=\frac{E_{cr}{}^{2}}{C_p\cdot p}\cdot\frac{\omega}{4\cdot\pi\cdot\ln\left(\frac{E_{cr}}{E}\right)} \qquad (14)$$

Substituting Eq.(14) in Eq.(11) and omitting in it the second term in the right part, we shall deduce expression for growth speed of the undercritical streamer

$$V_s=2\cdot\sqrt{D_e\cdot\frac{E_{cr}{}^{2}}{C_p\cdot p}\cdot\frac{\omega}{4\cdot\pi\cdot\ln\left(\frac{E_{cr}}{E}\right)}} \qquad (15)$$

Using usual estimations for free electron diffusion coefficient

$$D_e = \frac{10^5}{p_{Torr}}, \text{ cm}^2/\text{s} \quad (16)$$

and for critical value of electric field

$$E_{cr} = 30 \cdot p_{Torr}, \text{ V/cm} \quad (17)$$

it is easy to get the final expression for an estimation of growth speed of undercritical streamer discharge

$$V_s = \frac{3.6 \cdot 10^5}{\sqrt{\lambda_{cm} \cdot \ln\left(\frac{30 \cdot p_{Torr}}{E_{V/cm}}\right)}}, \text{ cm/s} \quad (18)$$

(λ - wavelength of MW radiation, cm).

Equation (18) specifies weak dependence of speed of distribution from undercriticality, an electric field and pressure until the field does not come nearer to critical. Thus growth rate of streamer unboundedly (within the framework of the given approach) grows, directing to the big values.

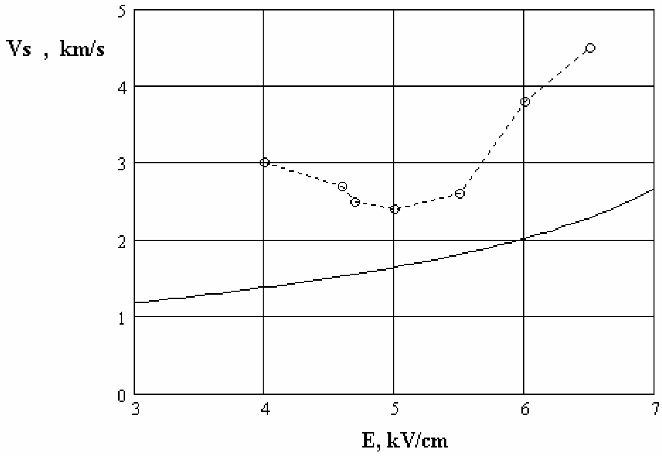


Figure 12. Dependence of growth speed of undercritical streamer on amplitude of an electric field, *p*=200 Torr, λ = 8.5cm. Eq.(15) - continuous line, experiment - dotted line with points.

So simplified approach does not apply for the strict description of the phenomenon, however, the Eq.(18) gives the values of the growth rate rather close to measured in various experiments of Ref.[10], that specifies a correct choice of the determining factor of development of undercritical discharge. On Fig.12 an estimation of the streamer growth rate Eq.(18), continuous line, is compared with the results of measurements borrowed of Ref.[10] for a case p=200 Torr, λ= 8.5cm, dotted line with points. It is visible Eq.(18) underestimates speed of propagation, though the general course of dependence is reproduced well.

Estimation Eq.(18) at constant value of undercriticality predicts full independence of speed on pressure and on a field. This result misses the data of the measurements[10] indicating some growth of speed with growth of pressure; however this divergence does not leave the limits of accuracy of measurements.

The radius of a head part of the streamer channel is determined by depth of front of ionization

$$a_s = 2 \cdot \sqrt{\frac{D_e}{\nu_i}} \quad (19)$$

where ionization frequency is defined by Eq.(14). Using known rates Eq.(16) and Eq.(17) it is simple to deduce the formula convenient for an operative estimation

$$a_s = 11.1 \cdot \frac{\sqrt{\lambda_{cm} \cdot \ln\left(\frac{30 \cdot p_{Torr}}{E_{V/cm}}\right)}}{p_{Torr}}, \text{ cm} \tag{20}$$

Dependence of radius of the channel on pressure of gas for concrete values of wavelength of radiation and intensity of a field is shown on Fig.13.

On Fig.12 it is visible, that typical value of speed of development of the streamer of undercritical discharge lies in a range of units of kilometers per second. For estimation of its value Eq.(18) can be used.

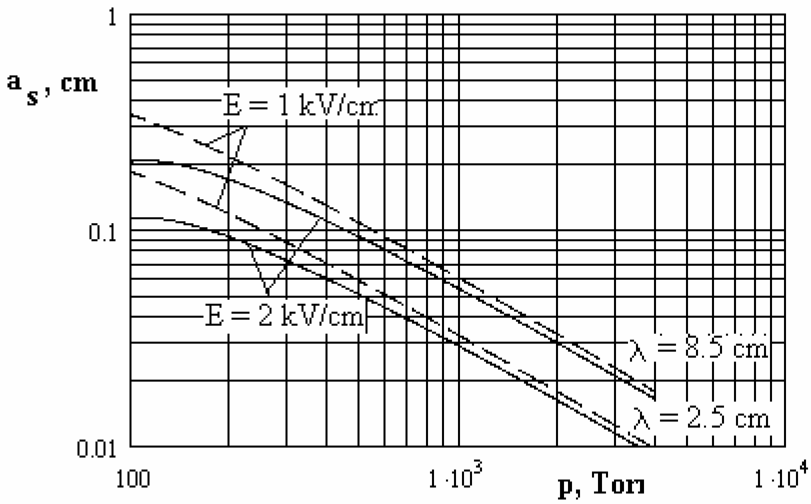


Figure 13. Estimation of radius of the streamer channel of undercritical MW discharge

V. Border of area of existence of free and attached undercritical streamer discharges

Proceeding from the developed representations, it is possible to derive estimation for the border, dividing areas of existence of freely extending and attached types of streamer undercritical discharges.

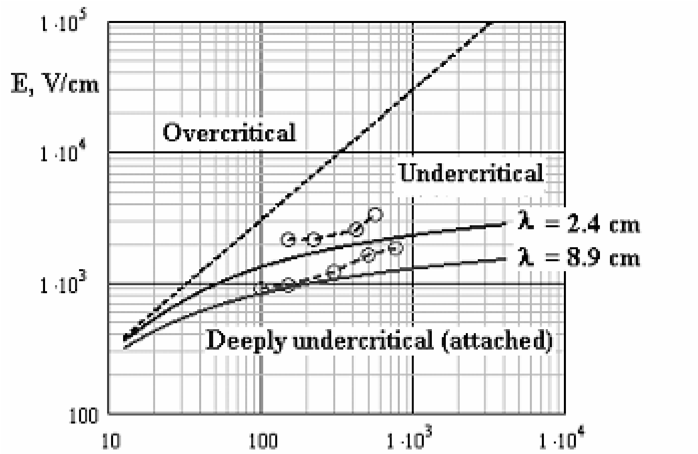


Figure 14. An estimation of the borders dividing areas of existence of the basic types of streamer discharges. Dashed lines with points - the data of measurements

The main property of a streamer - increasing of field amplitude on its head - is able to play role, if radius of a head part of streamer less than its characteristic length. If this condition is not satisfied, development of streamer stops. Having chosen as characteristic length size $\lambda/2\pi$, we shall deduce a condition for the border dividing free and attached undercritical types of discharges

$$\frac{\lambda}{4 \cdot a_s} = \frac{E_{cr}}{E_0} \tag{21}$$

Equation (20) together with Eq.(21) results in the transcendental equation

$$\frac{E_{V/cm} \cdot \sqrt{\lambda}}{\sqrt{\ln\left(\frac{30 \cdot p_{Torr}}{E_{V/cm}}\right)}} = 2 \cdot 10^3 \qquad (22)$$

Decisions of Eq.(22) for fixed wavelengths, used in experiments, are compared on Fig.14 with experimental data. Dashed lines with points are the data of measurements[11].

The estimated values are little bit lower measured. However, the general course of dependence on pressure and wave length quite corresponds to observations. It is also proves the advanced notions about streamer discharge.

VI. Summary

The resonant streamer nature of the freely propagating subcritical MW discharge is confirmed by direct modeling, which uses electron impact ionization, free electron diffusion, Ohm heating and electric field increase at tops of the streamer. Last property MW streamer can realize if the streamer periodically branching during development. The main channel with branches forms the analog of antenna system which sustains the resonant character during all process of discharge propagation. Such form of the streamer trails creates conditions for unlimited propagation of discharge in undercritical MW field. Modeling shows that system saves the resonant properties continuously. It results the high absorption ability of undercritical MW discharge, property, which is very important for many possible applications.

The fully developed discharge consists from many such elementary subsystems arising simultaneously and series creating complicated net of conducting hot channels.

Separate question is arising: why the main channel of the streamer creates the curved trajectory? Preliminary investigation shows that it is caused by instability of channel relatively to deviation of its direction if the proper electric field inducted on the streamer top much more than original electric field. But this question exits from frames of this work and will be studied later.

The branching is necessary factor needed for far propagation of a streamer discharge in undercritical MW field. Undercritical MW discharge is a nonlinear dissipative system. Each such system has ability to self-organization, to creation of structures. More deep understanding of MW discharge behavior demands farther development of theory, in particular, the defining of general parameter of optimization.

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