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Standing Sub-Compressed Detonation Wave in Hypersonic Combustible Mix Flow Sustained by Undercritical Microwave Discharge[*]

Kirill V. Khodataev[†]

UFSF “Moscow Radiotechnical Institute of RAS”, Russia

K.V.K@home.ptt.ru

This paper is devoted to theoretical study of detonation initiated in supersonic stream of air-propane mix at atmospheric pressure by microwave discharge pulse undercritical discharge. The designed numerical model of microwave discharge takes into account all main factors determining the properties of phenomenon. Plasma chemistry allows for ionization by electron impact, two-body and three-body attachment and recombination, molecular and electron excitation, dissociation etc. Plasma dynamics allows for transfer processes – gas viscosity, electron diffusion and thermal conductivity, and Ohm heating. It is one-liquid two-temperature model of combustible mix. Combustion is described by relax time depending on gas temperature and density. Electrodynamics takes into consideration the finite wavelength of radiation initiating and feeding the discharge, skin effect of inducted current at conditions of continuously changing electrical conductivity in discharge. The model uses real cross-sections of physical-chemical reactions, depending on calculated variables – density, gas temperature, electron temperature, and ionization and dissociation coefficients. Modeling confirms the microwave discharge ability to initiate detonation of stoichiometric mix of air and propane at speed of flow exceeding Chapmen-Jouguet value. This type of microwave discharge can be used for sustaining of standing plain or oblique stationary detonation wave in a hypersonic combustible flow at comparatively big cross-section. Pulse-periodical volumetric undercritical discharge with repetition frequency defined by size of averaged front depth of wave and flow speed is able to create periodically the net of ignition centers, which all together generate the common front. This stationary wave can be classified as sub-compressed detonation wave and thus can be sustained at wide diapason of incoming flow velocity exceeding Chapman-Jouguet velocity.

Nomenclature

E, H = effective amplitude of electric and magnetic field of microwave radiation
c = light velocity
ω, λ = microwave radiation frequency and wave length
σ = plasma electrical conductivity
ψ = ionization coefficient
ρ = density
T_e, T_g = electron and gas temperature
n_e, n = electron and gas number density
C_e, C_g = electron and gas thermal capacity at constant volume
τ_{eg} = electron-gas relaxation time
τ_h = chemical energy relaxation time
D_e = free electron diffusion coefficient
ν_i = sum of ionization, electron attachment and recombination frequencies
a_s = streamer radius
E_{cr} = critical value of electric field

I. Introduction

Preliminary it was estimated by numerical modeling that attached undercritical discharge is able the provocation of standing oblique detonation wave in stoichiometric mix of oxygen and hydrogen[1].

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[†] Professor, Head of Plasma Physics department, member AIAA

The microwave undercritical high-pressure discharges ability to ignite and to sustain combustion in flammable gas mixes in high-speedy flows (up to supersonic flows) is shown theoretically and experimentally. The performed theoretical study predicts that microwave discharges of the same type are able to ignite not only combustion, but detonation too. The filamentary structure of discharge creates the hot active centers like active points in cells in front of CJ detonation wave. It means that artificial active points, created by microwave radiation in definite region of combustible mix flow, will able to provoke the quasi-stationary weak detonation wave in fixed location. The stimulated weak detonation wave can exist at wide diapason of oncoming flow speed, if it is more CJ velocity.

Application of detonation regime of burning in propulsive systems is being discussed during more than half century. The most attractive idea is based on use of standing detonation wave in jet engines. Noted property of microwave undercritical discharge will help to solve some problems concerning to design of propulsive jet engines based on detonation in continuous mode.

II. Formulation of task

Experiments with MW discharges have shown that the energy release inside streamer channels during quite small time duration (in order of one microsecond and less) arises gas temperature inside the channel up to many thousands Kelvin (three and more). It was demonstrated by observing of evaporation of wolfram wire initiating the streamer undercritical MW discharge. Estimations and modeling on various numerical models have confirmed it too. The many experiments have been performed on ignition of combustion in combustible mixes, particularly in air-propane mix. It was shown that MW discharges are igniting combustion and increasing the speed of combustion[2,3,].

One from noted above simple numerical models has been used for study of possible ignition of detonation in the flammable gas mixes. The modeling had pointed on such principal opportunity[4,5].

The typical value of detonation front velocity in a still gas mix is about couple of km/s. In a flow with speed, which is more of this value, the ignition center generates the oblique detonation wave. In Ref.[[6]] as such initiating center the inlet cowl was used. At injection of hydrogen in to the flow before the cowl, the change of light in the channel was observed. It was interpreted as oblique detonation wave. Those experimental conditions were studied theoretically and numerical modeling confirms initiation of detonation[7].

The streamer MW discharge representing the complicated net of thermally exploding hot filaments is able to ignite the detonation in many points in transverse cross-section of engine channel. It means that in this case we will have the plane weak (underpressed) detonation wave attached to place of MW initiation. One can suppose that such regime can open the new opportunities for designing of scramjet based on use of standing detonation wave with short zone of combustion.

Here we investigate process of detonation ignition by a single streamer channel arising during individual discharge.

III. Numerical model

The model used in this investigation is described in Ref.[5]. Let us repeat the main physical assumptions been laid into basis of this model.

As it is known, discharge plasma generally is thermodynamically no equilibrium medium. Energy of electromagnetic field is absorbed first by electrons and then through elastic and not elastic collisions is transferred to a heavy component. Therefore, as a minimum, it is necessary to consider two-temperature model of plasma, distinguishing electron temperature and temperature of heavy component. The electronic temperature determines many constants of reactions. It determines constants in the equation of ionization balance, frequency of ionization, attachment, and recombination and so on. Electronic temperature determines time of a relaxation in a power exchange between electronic and heavy components, losses on recombination and bremsstrahlung radiation, active and reactive resistance of plasma. Dissociation of molecular gas depends, basically, from temperature of heavy components and described by Saha's equation. The description of thermal capacity, viscosity and heat conductivity of heavy components are based on assumption of its thermodynamic equilibrium, characterized by common temperature of heavy components. The heat conductivity, coefficient of diffusion and a thermal capacity of electronic component are defined in assumption about thermodynamic equilibrium of electronic component characterized by electron temperature. The propane presence at air in stoichiometric proportion changes gas-dynamic characteristics of air a little and its presence is taken into account by presence of chemical potential, relaxing into a heat with characteristic time of a relaxation, depending on density and temperature of heavy components. After passage of reaction of burning the number of molecules does not vary almost, that allows using the statement equation of air for the description of processes in air-propane mix. Dependence of time of a relaxation τ_h on the specified parameters is borrowed from Ref. [[8]]

$$t_{comb} = \frac{1.36 \cdot 10^{-10}}{(p, atm)^{3/2}} \cdot \exp\left(\frac{20000}{T, K}\right) \tag{1}$$

The system of the equations, put in a basis of model, consists from the equations of two-temperature one-speed gas dynamics with taking into account the relaxation of internal chemical energy:

- equation of ionization balance (equation of continuity for electron component) equation of continuity for

$$\frac{d\psi}{dt} = \nu_{iar} \cdot \psi + \nabla \cdot (D_e \nabla \psi), \tag{2}$$

- equation of power balance for electronic component

$$\frac{dT_e}{dt} = \frac{1}{C_e} \cdot \left(-T_e \nabla \cdot V + \nabla \cdot (\vartheta_e \nabla T_e) + \frac{T_g - T_e}{\tau_{eg}} + \frac{\sigma \cdot |E|^2}{n} \right), \tag{3}$$

- equation of power balance for heavy components

$$\frac{dT_g}{dt} = \frac{1}{C_g} \cdot \left(-T_g \nabla \cdot V + \nabla \cdot (\vartheta_g \nabla T_e) + \frac{T_e - T_g}{\tau_{eg}} + \frac{h}{\tau_h} \right), \tag{4}$$

- equation of movement heavy components

$$\frac{dV}{dt} = -\frac{1}{\rho} \cdot \nabla(nT_g + \psi n T_e), \tag{5}$$

- equation of continuity for heavy components

$$\frac{dn}{dt} = -n \nabla \cdot V \tag{6}$$

- equation of a relaxation of internal chemical energy *h*

$$\frac{dh}{dt} = -\frac{h}{\tau_h(p, T_g)} \tag{7}$$

Maxwell equations for electromagnetic field in assumption of monochromaticity

$$\nabla \times H = \left(\frac{4\pi\sigma}{c} - i\frac{\omega}{c} \right) \cdot E \tag{8}$$

$$\nabla \times E = -i\frac{\omega}{c} \cdot H \tag{9}$$

The system is completed by Ohm law and statement equations of electron and heavy component.

In those equations, the following functions are used:

$$\begin{aligned} \nu_{iar} &= \nu_i(\psi, n, T_e, T_g) + \nu_a(\psi, n, T_e, T_g) + \\ &+ \beta_r(\psi, n, T_e, T_g) n\psi \end{aligned}$$

- sum of ionization, attachment and recombination frequencies,

$$D_e = D_e(\psi, n, T_e, T_g)$$

- electron diffusion coefficient,

$$\vartheta_e = \vartheta_e(\psi, n, T_e, T_g)$$

$$\vartheta_g = \vartheta_g(\psi, n, T_e, T_g)$$

- thermal conductivity of electrons and gas,

$$\sigma = \sigma(\psi, n, T_e, T_g)$$

- electrical conductivity,

$$C_g = C_g(\psi, n, T_e, T_g)$$

- gas thermal capacity,

$$\tau_{ch} = \tau_{ch}(p, T_g)$$

- relaxation time of combustion, defined by Eq.(1),

$$\tau_{eg} = \tau_{eg}(n, T_g)$$

- relaxation time of electron in gas,

$$f_{dis} = f_{dis}(n, T_g)$$

- dissociation coefficient,

$$f = \psi\left(1 + f_{dis}(n, T_g)\right)^{-1}$$

- ionization coefficient.

All functions are based on real cross-sections of reactions.

Boundary conditions correspond to quite big radius of calculation area for gas perturbations, which have not a time for arrive to boundary and are transparent for MW radiation reflected by plasma channel. Initial conditions correspond to insignificantly small ionization and gas density decreased near the axis inside radius a = 0.03 cm up to critical value. The task is solved in 1D axis-symmetric approximation. Wavelength of MW radiation λ=8.9 cm equals to value used in real experiments. Air-propane mix is stoichiometric; its initial pressure is 1 atm at room temperature. Chemical potential of stoichiometric air-propane mix normalized on room temperature equals to $h_0 = 30.33$. Pulse duration of MW radiation is 0.6 μs.

IV. Result of modeling

The modeling confirms MW streamer discharge ability to ignite detonation in stoichiometric air-propane mix at atmospheric pressure.

After switching on of MW field, the initially insignificantly small ionization at future streamer channel starts to rise with high increment determined by degree of MW field supercriticality. Increase of electrical conductivity and characteristic radius of ionized channel is causing the rising of inducted MW current in channel (see Fig.1).

If electrical conductivity is high enough the electric field inside the channel decreases with clear displayed skin effect. The Fig.2(a) shows the electric field distribution at successive moments of time. The skin effect is seen on Fig.2 (b) too, where magnetic field amplitude distribution is presented. The spatial-temporal distributions of electric and magnetic field amplitude are presented in Fig.3. The external MW field is switched off after 0.6 μs.

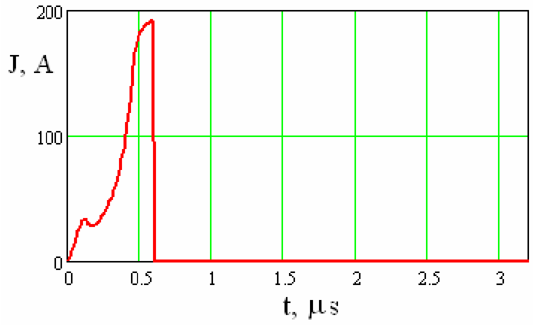
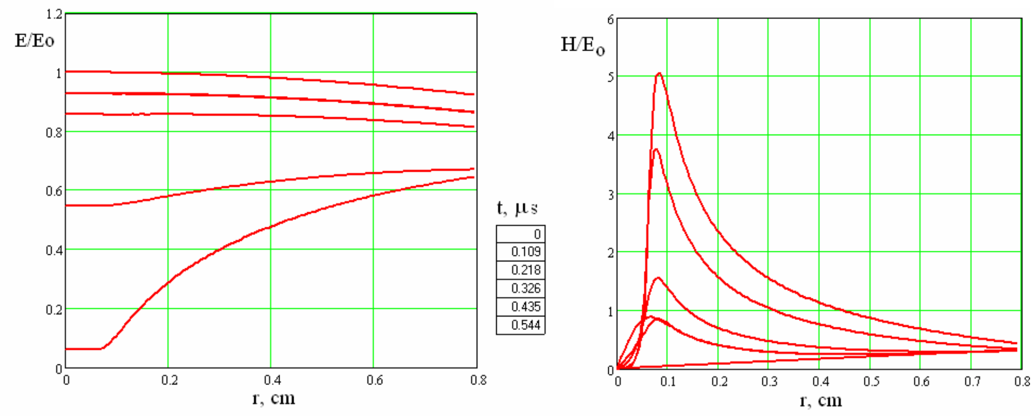


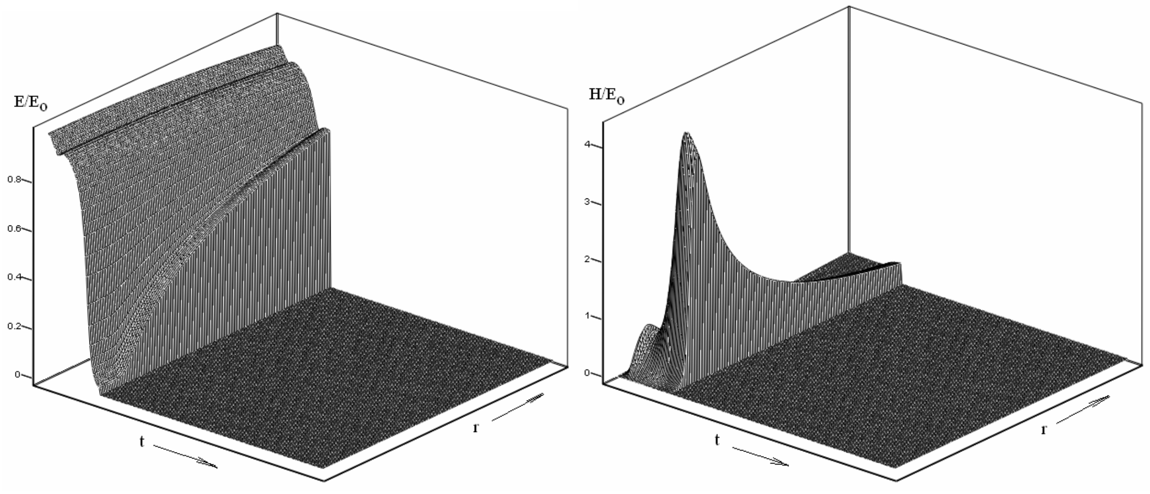
Figure 1. The amplitude of current inducted in developing streamer channel. τ_{MW} = 0.6 μs



(a) **(b)**

Fig.2. Electric and magnetic field amplitude distribution at successive moments of time.

Electric current in gas with finite electric conductivity naturally causes the Ohm heating. When electrical conductivity achieves the value corresponding to equality of skin-layer and the channel radius, Ohm heating is maximum because electrical field do not be decreased else and conductivity is high enough already. At this moment the gas temperature increases up to high value so quickly that gas density do not have time for decrease.

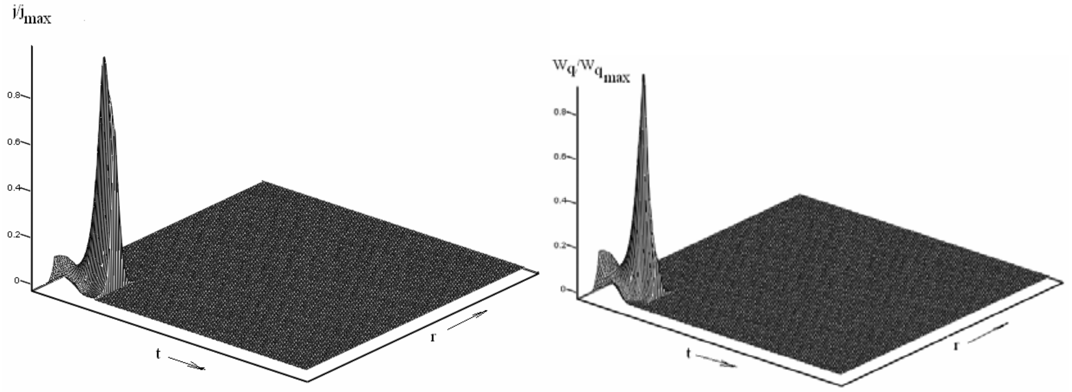


(a) **(b)**

Figure 3. Spatial-temporal distribution of MW field amplitude. (a) - electric field, (b) – magnetic field

Therefore, the chemical reaction time is quite small and detonation wave appears. Fast heating of gas generates high temperature and initiate both heat explosion and chemical reaction of combustion. Spatial-temporal distributions of MW current density and specific power of Ohm heating are shown on Fig.4. As result, the detonation wave with constant radial velocity equaled to Chapman-Jouguet value is appearing. The Ohm heating stops, when MW pulse is over, but detonation wave is propagating continuously up to boundary of calculated area. It is well illustrated by Fig.5, which demonstrates the temporary evolution of gas pressure and radial velocity.

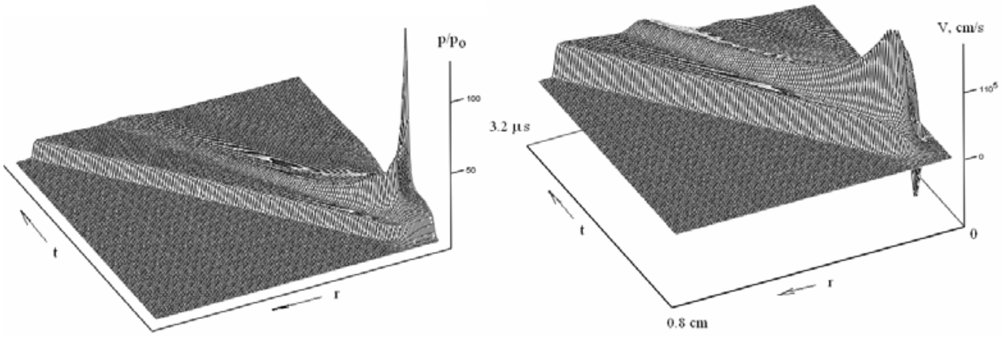
The ionization in channel continues to rise so skin layer become less than channel radius. The high current density in skin –layer caused a very intensive heat of gas in layer (see Fig.4 (a) and (b)).



(a) **(b)**

Figure 4. Spatial-temporal distributions: (a) – current density j/j_{max}, (b) – specific power of Ohm heating W_q/W_{qmax}

Such intensive heating creates secondary shock wave beside detonation wave. It is well seen in Fig.5.



(a) (b)

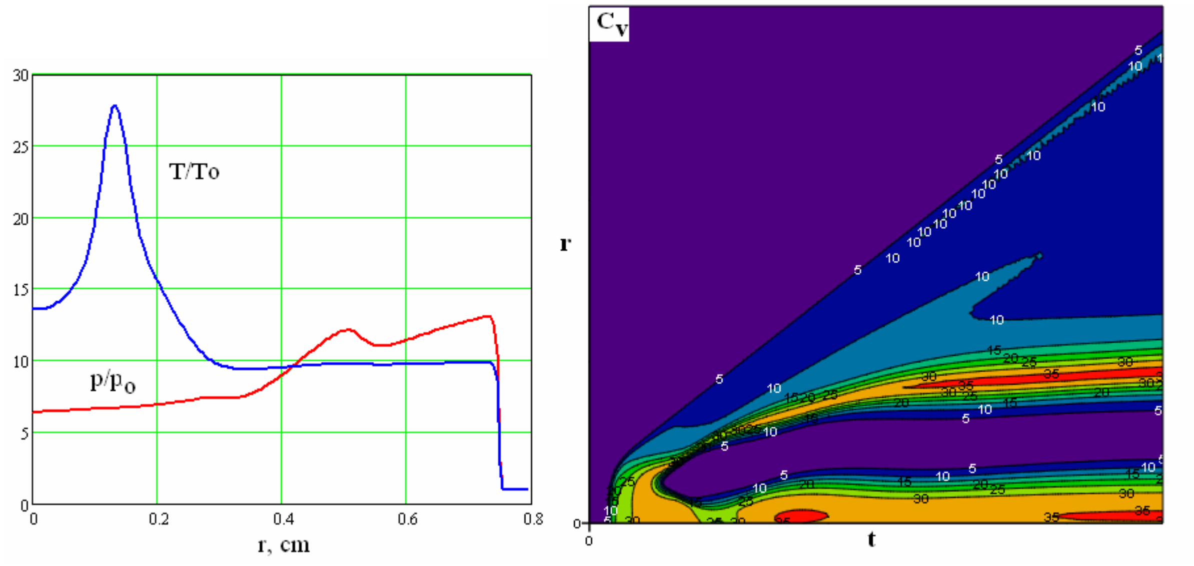
Figure 5. Spatial-temporal distributions: (a) – gas pressure p/p_0, (b) –radial velocity V, cm/s

The distributions of gas parameters behind the detonation front shown in Fig.6(a) are typical for theoretical model of Zeldovich et al. Temperature behind the front is approximately 3000 K. The observed speed of detonation front is 1.58 km/s. This value well corresponds to theoretical value for CJ wave for adiabatic parameter behind front $\gamma_2 = 1.2$:

$$M_{CJ} = \frac{\sqrt{(\gamma_2 - 1)\left((\gamma_2 - 1)h + \frac{(\gamma_2 + \gamma_1)}{\gamma_1 - 1}\right)} + \sqrt{(\gamma_2 + 1)\left((\gamma_2 - 1)h + \frac{(\gamma_2 - \gamma_1)}{\gamma_1 - 1}\right)}}{\sqrt{2\gamma_1}}, \qquad (10)$$

where h is chemical potential, normalized on T_1, indexes 1 and 2 are related to media before and after front.

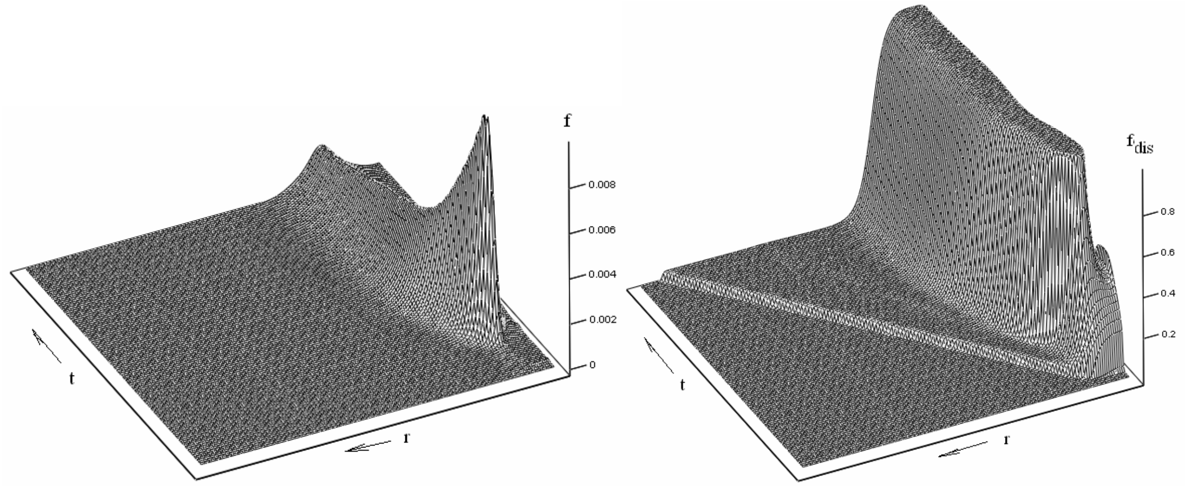
Thermal capacity of mix just behind front equals to ~5 (see Fig.6(b)). It corresponds to above-mentioned value of γ_2.



(a) (b)

Fig.6. Radial distributions of gas temperature and pressure at moment of time t = 1.82 μs - (a); distribution of mix thermal capacity – (b)

Increase of thermal capacity is defined by expenditures on dissociation and molecular excitation. Dissociation coefficient behind detonation front equals to ~0.05. Ionization coefficient has maximum on the axis of the channel only at first moment and later equals approximately to 10^{-3} at radius of skin layer in hot channel created by discharge and has a very small value behind the detonation front (see Fig.7).



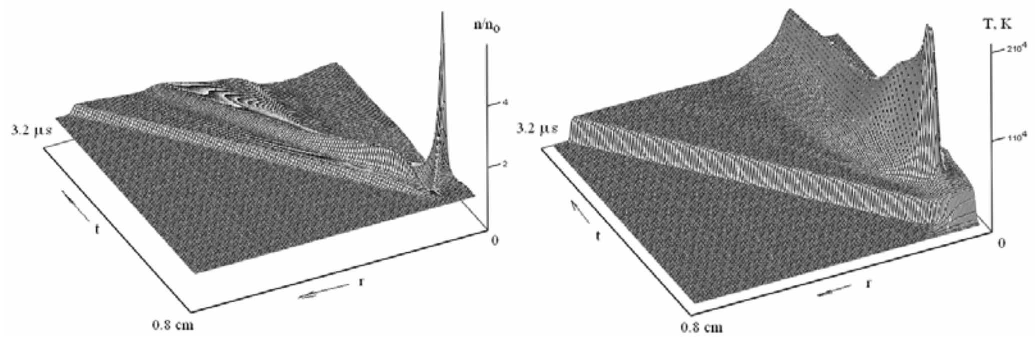
(a) **(b)**

Figure 7. Spatial-temporal distributions: (a) – ionization coefficient, (b) – dissociation coefficient

Electron temperature differs from gas temperature (much more) only during MW pulse. After end of MW pulse, the electron temperature equals to gas temperature. Note, that at axis the hot channel with initially small radius is saved. Interesting, that mix temperature is maximum not at the axis but at radius of skin layer. It is seen in Fig.8, where spatial-temporal distributions of gas density and temperature are presented.

V. Discussion

Thus, calculations have shown that each elementary filament of a MW streamer discharge is able to ignite detonation in a gas combustible mix. Average distance separating the filaments is of the order of quarter of wavelength (more or less in dependence on discharge conditions). It means that all gas fulfilled the discharge region will have been combusted through approximately 5-10 μs and detonated area will increase with CJ speed. Characteristic size of a typical MW streamer discharge is comparable with several wavelength of MW radiation. Earlier it was proved theoretically and experimentally that streamer discharge can be sustained in a high-speed velocity of a gas flow.



(a) **(b)**

Figure 8. Spatial-temporal distributions: (a) – mix density, (b) –mix temperature

It is understandable, because the propagation velocity of undercritical streamer MW discharge is of the order of several kilometers per second. Consequently, behind the layer of MW discharge representing the continuously reproduced net of streamers all combustible gas mix must be detonated. This standing front of forced detonation can be created in the flow velocity, which is more than velocity of Chapmen –Jouguet (M>4.5). This regime one can classify as underpressed (or weak) detonation wave.

It is natural to suppose that the using of such regime of burning can help to design the scheme of scramjet with very high specific impulse. Using of ordinary rates for detonation and shock waves and for nozzles, one can prove that in the simplest scheme of scramjet (without inlet diffuser and with outlet nozzle) by means of the standing weak detonation wave the specific impulse in several times more than in usual scramjet with supersonic combustion can be gotten.

It is needed to add, analyzing the process of MW streamer development, one must necessarily take into account the skin effect, which plays important role, influencing on streamer parameters, its general properties and ability to ignite combustion or detonation.

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