Various types of initiators for attached undercritical MW discharge ignition*

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The most favorite type of initiators of attached undercritical microwave discharge is the passive half-wavelength resonant vibrator. This type of initiator being oriented along a gas flow and electric field of microwave radiation is very usable for attached discharge ignition in a speedy flow including supersonic one. The electric field increase at vibrator’s tops is about ratio of wavelength to vibrator diameter. It means that discharge can be ignited in microwave field which amplitude is less than critical value in number of times equaled this ratio. Tandem of two half-wavelength passive vibrators being located along electric field on one axis and separated by a gap, which is comparative with its diameter, is able to increase the field amplitude inside the gap in two times more than one vibrator. The same result can be achieved if a half-wavelength vibrator is curved into a circle with small gap between its ends. Noted types of vibrators can be united into a system of vibrators for creation many active points in gas flow in volume or at surface of streamlined body. Properties of described types of initiators are studied numerically in detail. Investigated new types of igniters broaden the area of possible applications of attached microwave discharges particularly for combustion ignition in high-speed fuel mix flow.

Nomenclature

\[ E = \text{effective amplitude of electric field of microwave radiation} \]
\[ E_{cr} = \text{critical value of electric field} \]
\[ I = \text{amplitude of inducted current} \]
\[ c = \text{light velocity, cm/s} \]
\[ \omega = \text{microwave radiation frequency, s}^{-1} \]
\[ \lambda = \text{microwave radiation wavelength, cm} \]
\[ \sigma = \text{electrical conductivity, s}^{-1} \]
\[ N = \text{total number of vibrators in system} \]
\[ L_v = \text{length of vibrator, cm} \]
\[ R_s = \text{main radius of loop-shaped vibrator, cm} \]
\[ k = \frac{2 \pi}{\lambda} - \text{wave number, cm}^{-1} \]
\[ l = \text{length along conducting channels, cm} \]
\[ a = \text{radius of initiator, cm} \]
\[ p = \text{gas pressure, atm} \]
\[ i = (-1)^{1/2} \]

I. Introduction

Creation and sustaining of discharge in air at pressure compared with atmospheric by microwave (MW) radiation without means of initiation (so named overcritical MW discharge) demands too high level of specific power of radiation (of order 1 MW/cm²). To achieve such high values in a focus of MW radiation with minimum cross-section \( \lambda^2/2 \), defining by diffraction limit, the needed full power of radiation source is:

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where $p$ – gas pressure, atm, $\lambda$ - wavelength of radiation, cm. For $p = 1$ atm and $\lambda = 10$ cm generator with power 0.6 GW is required for breakdown of air. Design of generators with so high power is possible, of course, for physical special experiments, but in usual conditions it is inapplicable. Thus, the overcritical MW discharges have not a perspective for practical applications.

Many experimental works have shown that gas discharge can be created in MW radiation at specific power, which can be significantly (several orders of magnitude) less than critical value, used in Eq. (1). It is so named undercritical discharges. Low level of MW generator power, needed for discharge creation, does it much more attractive for various applications. Undercritical discharges at gas pressure more 0.1 atmosphere have a streamer character; they represent a complicated net of electrically conducting hot channels. This net is excellence absorber of radiation and can be used for in aerodynamics itself for flow-field influence, for combustible mix volumetrical ignition and so on. The undercritical MW discharges can be created by means of initiation. The most effective known kind of initiators is metallic sharp object, which is located in focus of MW radiation. At small vicinity of sharp tip of the metallic object amplitude of electric field is much more than initial value (without initiator) and can exceed the critical value. In this case the breakdown takes place on surface of sharp tip and streamer discharge begin propagating far away from initiator because the its sharp head carries with itself the field more than critical value although the field around is much less than critical value. It is free undercritical MW discharge.

Experiments show that if amplitude of electric field of radiation is less than some value (order of 1-2 kV/cm almost independently on air pressure) the streamer after breakdown can’t go farther than part of wavelength. Streamers remain attached to tip of initiator. It is so named attached undercritical discharge (or deeply undercritical discharge). Classification of described types of discharge and domains of them existence is developed in Ref.[1,2,3,4,5,6].

Attached undercritical discharges exists at the shortest focus of radiation created by generator with power, which can be estimated by Eq. (2)

$$P_{\text{attached}} \leq 1.1 \cdot 10^3 \cdot \frac{\lambda^2}{2}, W$$

Note, that Eq. (1) is enable for draft estimation only; more detail the boundary between volumetric and attached discharges is studied in Ref.[7].

Both types of initiated undercritical discharge (volumetric and attached) are able to be sustained in speedy flow (up to supersonic flow).

Paper is devoted to study of various types of initiator, which are used in experiments with attached discharge initiation and are able to be useful in some applications.

II. Method of microwave field calculation

The requirements to initiator can be formulated by following items. Initiator must:

- increase the electric field amplitude locally in needed times (up to critical value),
- being placed into flow be quite streamlined,
- in frame of system of initiators be able to work effectively together with other electromagnetically connected initiators.

For design of the initiator and initiator system it is necessary to be able to calculate the field distribution near initiators. For this purpose the approach, described firstly in Ref.[8] and more in detail in Ref.[4], was used. Designed method is based on integral equation for current, inducted in arbitrary dislocated in space thin electrically conducting channels

$$J(l) = \mathcal{G}(l) \left[ E_0(\vec{r}(l)) \frac{dl}{dl} \right] +$$

$$i \frac{k}{c} \mathcal{G}(l) \left[ J(l') \left[ G(\vec{r}(l),\vec{r}(l')) \left( \frac{dl}{dl'} \cdot \frac{dl}{dl} \right) + \frac{1}{k^2} \left( \frac{dl}{dl} \cdot \nabla \nabla (r, r(l')) \right) \right] \right] dl'$$.  

(3)

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where

\[ G(\vec{r},\vec{r'}) = \frac{\exp(ikR(\vec{r},\vec{r'}))}{R(\vec{r},\vec{r'})} \]  

(4)

\[ R(\vec{r},\vec{r'}) = \sqrt{a(l)^2 + |\vec{r} - \vec{r'}|^2} \]  

(5)

\[ \mathcal{G}(l) = \pi \cdot a^2 \cdot \sigma(l), \]  

(6)

\[ k = \frac{2\pi}{\lambda}, \]  

(7)

\[ \vec{E}_o(\vec{r}) \] - vector of electrical field amplitude of MW radiation, do not perturbed by conducting channels.

Calculated current defines the field space distribution by known operators:

\[ \vec{E}(\vec{r}) = \vec{E}_o(\vec{r}) + \frac{k}{c} \cdot \int_{l'} \left[ J(l') \cdot \nabla \cdot - \nabla \cdot \frac{\partial}{\partial l'} \left( \vec{G}(\vec{r},\vec{r'}(l')) \cdot \nabla \cdot \vec{G}(\vec{r},\vec{r'}(l')) \right) \right] dl' \]  

(8)

Coordinates of conductors channels are defined parametrically

\[ \vec{r} = \vec{r}(l) \]

These equations were successfully used for modeling of MW streamer development. Equation [3] need not artificial boundary conditions against to system of Maxwell’s equations (or them Helmholtz modification) and requires much less calculation time.

### III. Single initiator

The simplest initiator is metallic cylinder with rounded tips as one can see on Fig. 1. If the length of cylinder is comparable with half wavelength it possesses resonant properties. It is electromagnetic vibrator.

![Figure 1. The symplest initiator – symmetrical metal cylinder, straight electromagnetic vibrator](image)

![Figure 2. Electric field amplitude distribution in vicinity of the vibrator with resonant length. \( \lambda = 12.33 \text{ cm}, L_v = 5.5 \text{ cm}, a= 0.1 \text{ cm} \)](image)
This vibrator’s configuration is symmetrical. The vibrator, being placed into radiation field with linear polarization parallel to electric field, has equal maximum induced field on both tops. One can see it in Fig. 2, where electric field amplitude distribution in vicinity of the vibrator is shown.

Main parameter of initiator is coefficient of electric field increase

\[ \eta \equiv \frac{E_{\text{top}}}{E_0}, \]

where \( E_{\text{top}} \) – maximum field on vibrator’s top, \( E_0 \) – field amplitude of MW radiation in place of vibrator location. On Fig. 3 the coefficient \( \eta \) dependence on vibrator length \( L_v \) is represented. One can see 1st, 2nd and 3rd resonant mode, which correspond to vibrator’s length approximately equaled to 1, 2 and 3 half wavelength of MW radiation. Usually the 1st resonance is used

![Figure 3. Coefficient \( \eta \) dependence on vibrator length \( L_v \)](image)

Results of theoretical and experimental study of the field increase coefficient \( \eta \) give the estimation for vibrator with resonant length

\[ \eta \equiv \frac{E_{\text{top}}}{E_0} \approx \frac{\lambda}{4 \cdot a}, \quad (a << \lambda), \]  \[ (9) \]

where \( a \) – curvature radius of vibrator top. This type of discharge was successfully used in experiments on study of physics of undercritical MW discharges in various conditions (see for example Ref.[2,10,11]).

Often it is needed to have initiated discharge only on one top of initiator. In this case the vibrator with one sharp top is used. Other top is blunt, as it is shown on Fig. 4.

![Figure 4. Electromagnetic vibrator with sharp and blunt tops.](image)

Because the increase coefficient is reverse proportional to curvature radius of top the semi-sharp vibrator has maximum field only on one top (see Fig. 5, where electric field distribution is presented for resonant semi-sharp vibrator). Resonant curve of semi-sharp vibrator is shown on Fig. 6.

The semi-sharp vibrator was successfully used in many plasma aerodynamics and combustion experiments[2,12].

It is nonsense to use initiators with minimum curvature radius \( a_{\text{min}} \) which is less than diffusion length of attachment \( L_d \). In general case the field needed for breakdown at initiation by means of resonant electromagnetic vibrator is estimated by formula

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\[ E_0 > E_{cr}(p) \cdot \frac{4 \cdot (a_{\text{min}} + l_w)}{\lambda}, \]  

where diffusion length of electron attachment can be estimated by expression \(^{13}\)

\[ l_w \approx 10^{-3} \cdot p, \text{ cm. (11)} \]

More over, at initiated attached discharge presence the density of inducted current on a sharp top surface can be too high at small curvature radius. It results the top ablation through evaporation during burning of discharge. Taking into account this obstacle in case, when the very high field increase coefficient is needed, one can use combination of two resonant vibrators located on one straight line parallel to electric field with small gap between there sharp tops, as it is shown in Fig. 7. Such combination allows increasing of coefficient \( \eta \) in two times.

Field distribution around combination of two resonant vibrators is shown in Fig. 8. Maximum field amplitude is observed in gap between the nearest tops of vibrators. Figure 9 demonstrates dependence of coefficient \( \eta \) on vibrator length for different values of gap size. Increase coefficient arises in two times approximately if the gap equals to diameter of cylinder (in general case – \( 2a_{\text{top}} \)).
Figure 8. Electric field amplitude distribution around two resonant vibrators placed in MW radiation with wave length $\lambda = 12.33$ cm.

Figure 9. Dependence of coefficient $\eta$ on vibrator length for different values of the gap size $2\Delta$.

Idea of field doubling in gap between tops of vibrators can be realized not only by combination of two straightforward vibrators but by means of single resonant vibrator curved into ring with gap between its ends, how one can see on Fig. 10. Variant (b) in Fig. 10 differs from variant (a) by straightforward parts of vibrator’s ends. Those parts are comfortable for setting of vibrator to insulator plate for initiation MW attached discharge on its external surface.

Calculations showed that field increase coefficient is arising more than two times (up to four times and more). Special study of problem gave the explanation of this phenomenon. Additional increase is explained by decrease of radiation resistance of curved vibrator. Radiation resistance, determining the power of reradiating of passive vibrator, placed in external MW field, is defined by Eq. (12).

Figure 10. Resonant vibrator curved into ring with small gap between its tops.
where \( |J|_{\text{max}} \) - maximum current amplitude in distribution along vibrator. If small radius of vibrator is constant along vibrator, the current amplitude distribution is near to cosinusoidal with zero at its ends. Radiation power is numerator in Eq. (12). Figure 11 demonstrates the calculated dependence of radiation resistance \( R_w \) on distance \( \Delta \) between ends of curved vibrator, divided on its length \( L_v \), for case of resonance (for variant (a) in Fig. 10). At \( \Delta = L_v \) the vibrator is the usual straightforward half-wavelength vibrator with well known magnitude of radiation resistance \( R_w = 75 \Omega \). But the less rate \( \Delta/L_v \) (the less radius of curvature), the less radiation resistance.

Numerically calculated dependence can be satisfactory approximated by simple formula Eq. (13)

\[
R_w \approx \sqrt{10^7 + \left(72 \cdot \frac{\Delta}{L_v}\right)^2}.
\]  

Figure 12. Dependence of quality factor on distance between tops of curved vibrator with resonant length \( L_v \) calculated for variant (a) in Fig. 10. Solid line with circuls – estimation Eq. (14), black cases – numerical experiment.
Losses on reradiating are limiting the inducted current at resonance and quality factor of resonant vibrator, which can be estimated by expression (14)

\[ Q \approx \frac{2\pi \cdot \ln \left( \frac{1}{ka} \right)}{c \cdot R_w} \tag{14} \]

The estimation Eq. (14) based on data in Fig. 11 is compared with numerical modeling data in Fig. 12. One can see a good coincidence.

Maximum field amplitude is created in gap between tops of loop-shaped vibrator with \( \Delta L < L_w \). One can see it in Fig. 13, which demonstrates spatial distribution of electric field amplitude around the loop-shaped vibrator for both variants, (a) and (b) in Fig. 10. The magnitudes of increase coefficient \( \eta \) for real constructions exceed 2-3 hundred at small radius of vibrator \( a = 0.05\text{cm} \) and main radius of loop-shaped vibrator equaled to \( \sim 0.9-1 \text{ cm} \). Calculations are executed for wavelength of MW radiation \( \lambda = 12.33 \text{ cm} \). But it must be noted, that field distribution, field increase coefficient, quality factor etc. depend only on relation between vibrator sizes and wavelength.

![Figure 13. Electric field on the axis of and maximum field at ends of cylindrical passive vibrator via electrical length of it. Radius of vibrator \( a=0.1/k \). \( \lambda = 12.33 \text{ cm} \)](image)

![Figure 14. Resonant curves for inducted maximum current \( |J|_{\text{max}} \) for variants (a) and (b)](image)

Field increase by loop-shaped vibrator is very sensitive concerning to values of its main radius \( R \) at given wavelength. It is seen on Fig. 14, where resonant curves for maximum inducted current \( |J|_{\text{max}} \) are represented for variants (a) and (b) in Fig. 13. If we want to work in resonant regime, relative error in loop radius magnitude must be less than reverse quality factor \( Q \). The needed precision tuning of such vibrators is a serious unpleasant problem.

In experiments with combustible gas mix injection into domain of attached MW discharge the hollow tube half-wavelength vibrator with many tops on one side was often used\(^{14} \). Design of this type of vibrator with six tops and electrical field distribution in cross plane corresponding to vibrator end with sharp tops are represented by Fig. 15. Radius of the tube must be less than its length, which is approximately equals to half-wavelength of MW radiation.
Figure 15. Cylindrical hollow vibrator with many sharp tips (left) and MW field distribution in the plane of sharp tips (write). $R_s = 0.6$ cm, $L_v = 4.2$ cm, $N = 6$, $\lambda = 12.33$ cm

It is supposed that such hollow vibrator creates attached discharge with radius of order of tube radius.

IV. System of vibrators

In some aerodynamic and combustion experiments the system of attached MW discharges, simultaneously working in given several points, can be required. The example of such system is shown in Fig. 16. Many semi-sharp vibrators, the same as in Fig. 4, are supported by common pylon, which can be metal. Such construction can be located in a gas flow tunnel, as it is shown in Fig. 16 (right). Of course interior of the tunnel must be open for MW radiation feeding the vibrators.

Figure 16. Plane system of straight semi-sharp vibrators (left). Example of this system use in a gas flow tunnel.

Examples of electric field amplitude distribution for system of six semi-shape vibrators are shown in Fig. 17 for distance between vibrators $d = 1$ cm (as in Fig. 16), and $d = 4$ cm.

Figure 17. Electric field amplitude distribution for system of six semi-shape vibrators. (a) – $d = 1$ cm, (b) – $d = 4$ cm
Note, that instead of six maximums of field, corresponding to six sharp tops of vibrators, one can see only four maximums (Fig. 17(a)). The matter in that the electrodynamical properties of such system depend on degree of communication between vibrators, which is determined by distance between vibrators. If the distance is large, more wavelength of MW radiation, each vibrator is working independently without influence one on another. But if the distance is small, as in Fig. 16, all vibrators strongly communicate, so inducted currents in each vibrator depend on currents induced in all other vibrators. The distribution of electric field amplitude on sharp top between vibrators strongly depends on vibrators tuning. Figure 18 demonstrates this dependence for system of six vibrators similar to shown in Fig. 16 for distance between vibrators equaled to 20cm and 1 cm, excited by plane wave propagating normally to plane of vibrator system with electric field parallel to axis of vibrators.

Figure 18. Distribution of electric field amplitude on top of vibrator on series number of vibrator for system similar to shown in Fig. 16, $\lambda = 12.33 \text{ cm}$, $N = 6$. (a) – $d = 20\text{ cm}$, (b) – $d = 1\text{ cm}$.

Because the communicated vibrators represent itself by the distributed oscillatory system the resonant excitation can be realized as eigenmodes with distribution of electric field amplitude at tops of vibrators approximately described by Eq. (15)

$$E_{\text{top}}(n) \sim \cos\left(2\pi \cdot \frac{n \cdot (N-l)}{N-l} \cdot m\right), \quad (1 \leq n \leq N), \quad \left(1 \leq m \leq \frac{N}{2}\right),$$

(15)

where $N$ – total number of vibrators, $n$ – series number of vibrator, $m$ – mode number. In Fig. 18(b) the highest modes are not seen because them quality factor is small and conditions of excitation are not optimal.

Figure 19. Circular system of straight semi-sharp vibrators (feebly connected system). (a) - system itself, (b) - system in a gas flow tunnel.
The vibrators system can be circular or some other needed configuration. Example of circular system of vibrators with comparative weakly communication is presented on Fig. 19.

Some aerodynamic experiments need the creation of discharges on surface of streamlined body\textsuperscript{15}. For this purpose the loop-shaped vibrator shown on Fig. 10(b) can be used. In concrete experiment formulation the row of separate discharges must be located on a surface transverse to flow velocity with distance between discharges \( d = 0.5 \) cm. The designed system of loop-shaped vibrators satisfying this condition is shown in Fig. 20. The upper surface of dielectric plate, which is supporting the vibrators, will serve as part of main surface. The vibrators are located inside the streamlined body (wing or blade), as it is shown on Fig. 21(a).

![Figure 20. System of loop-shaped vibrators supported by dielectric plate (strong connection)](image)

![Figure 21. Experimental wing with introduced loop-shaped vibrator system for MW discharges ignition in gaps of vibrators. (a) – view of wing interior, (b) – general view of composition wing – MW horn.](image)

The MW feeding of vibrators will executed by means of waveguide and fan horn (see Fig. 21(b)). The MW discharges between each pair of vibrator tops will create the active points of energy addition for influence on properties of boundary layer and flow-field\textsuperscript{15}.

Above it was noted that loop-shaped vibrator has a small radiation resistance and correspondingly high quality factor. This circumstance together with required small distance between vibrators results very strong communication. So it is not surprised that this system has a very high sensitivity to tuning of each vibrator (to ratio of vibrator perimeter length to wavelength of radiation). Dependence of gap field on series number of vibrator and on them perimeter length for system of 10 vibrators is showed in Fig.22. One can see the strong sensitivity of gap field distribution from perimeter length.

Thus this system needs the means of tuning. It is supposed to solve this problem by insertion of changing cylinders with high dielectric permittivity into loops.

Calculations show that this sensitivity is decreasing if quality factor of system is decreasing. It means that just if at first moment of switching-on MW radiation the gas breakdown will take place only at some gaps, where the field is maximal, later during the discharge development the field distribution will being changed, causing the breakdown in neighbor gapes. Possibly it will give a homogeneous burning of discharges along system gaps.

But it is the problem for next step of investigation.
Figure 22. Gap field distribution between loop-shaped vibrators in dependence on its main radius. \( \lambda = 12.33 \, \text{cm}, N=10, d=0.5 \, \text{cm} \).

V. Summary

For MW discharge ignition one can recommend

- in gas free flow - straight semi-sharp resonant vibrator, located in flow, for ignition in a volume
- near surfaces - loop-shaped resonant vibrator, located under surface outside the flow, for discharge ignition near surfaces

Both types of vibrators can be used in composition of systems of various configurations (plane, circular etc.). Vibrators in system are connected between themselves so systems need a precision tuning for obtaining of required field distribution, especially, the systems of loop-shaped vibrators.

VI. Acknowledgments

The work is performed with financial support of EOARD, (Projects ISTC # 2429p and #2820p and Project CRDF # UKE2-1508A-KV-05)

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