47th AIAA Aerospace Sciences Meeting and Exhibition
5-8 January 2009, Orlando, Florida

AIAA 2009-1410

Microwave breakdown threshold at low and high pressure

Kirill V. Khodataev*
Moscow Radiotechnical Institute of RAS, Russia, K.V.K@home.ptt.ru

The critical value of MW field amplitude for gas breakdown is studied theoretically in a wide diapason of pressure. It is confirmed that at low pressure the breakdown takes place if well known effective magnitude of MW electric field equals to critical value of DC electrical field. But at high pressure the breakdown threshold corresponds to equality of MW electric field amplitude to critical value of DC electrical field. The simple formulas are recommended for estimations of breakdown MW field amplitude in air at full pressure diapason

Nomenclature

E	=	amplitude of electric field
E_0	=	amplitude of electric field of microwave radiation
E_{cr}	=	critical value of electric field amplitude
c	=	light velocity
ω, λ	=	microwave radiation circular frequency and wave length
k	=	$2\pi/\lambda$ - wave number
p	=	air pressure, Torr
e, m	=	electric charge and mass of electron
V	=	electron velocity
T_e	=	electron temperature
$1/\tau_{tr}$	=	frequency of electron transport collisions
τ_{rel}	=	electron energy relaxation time in a gas
ν_a	=	attachment frequency
E_{DCcr}	=	DC critical electric field
t	=	time

I. Introduction

The breakdown in the high frequency field has features that require specifications since MW discharge properties essentially change with respect to pressure. Many now classical works have been devoted to investigations of the breakdown process[1,2,3]. Most of them were devoted to gas breakdown in gases of low and moderate pressure. Features of breakdown in microwave radiation at high gas pressure have required additional investigation.

II. The simplified theory

Here we will develop a simplified gas breakdown theory in wide gas pressure range. Let us consider a behavior of electrons in a gas at presence of a periodic electric field

$$E = E_0 \cdot \cos(\omega t) \tag{1}$$

The equation of individual electron motion in the electric field can be written in the form

$$\frac{dV}{dt} + \frac{V}{\tau_{tr}} = \frac{eE_0}{m} \cdot \cos(\omega t), \tag{2}$$

where
e, m are the charge and mass of the electron,
$1/\tau_{tr}$ is a frequency of transport collisions.

* Professor, Head of Plasma Physics department, member AIAA

The second term in the left side of the equation (2) describes the momentum loss of the electron in the transport collisions.
The equation (5.2) is the linear differential equation of the 1-st order. Its solution can be written in the form

$$V = \exp\left(-\frac{t}{\tau_{tr}}\right) \cdot \int \frac{e \cdot E_0}{m} \cdot \cos(\omega t) \cdot \exp\left(\frac{t}{\tau_{tr}}\right) dt \,,$$

After integration one has an equation

$$V = \frac{eE_0 \tau_{tr}}{m} \cdot \frac{\cos(\omega t) + \omega\tau_{tr} \cdot \sin(\omega t)}{1 + (\omega\tau_{tr})^2} . \tag{3}$$

A motion in electric field **E** with the velocity **v** is accompanied by a gain of energy, which is accounted in the right hand side of the electron energy balance equation

$$\frac{dT_e}{dt} + \frac{T_e}{\tau_{rel}} = \frac{e^2 E_0{}^2 \tau_{tr}}{m \cdot 2} \cdot \frac{(1 + \cos(2\omega t) + \omega\tau_{tr} \sin 2(\omega t))}{1 + (\omega\tau_{tr})^2} , \tag{4}$$

where
T_e is electron temperature,
τ_{rel} is relaxation time of electrons ,

For simplicity we consider that the gas temperature is negligibly small with respect to the electron temperature.

The Eq.(4) can be also integrated. Its solution is

$$T_e = \frac{e^2 E_0{}^2 \tau_{tr}}{m} \cdot \frac{1}{2\left(1 + (\omega\tau_{tr})^2\right)} \exp\left(-\frac{t}{\tau_{rel}}\right) \int (1 + \cos(2\omega t) + \omega\tau_{tr} \sin 2(\omega t)) \exp\left(\frac{t}{\tau_{rel}}\right) dt \tag{5}$$

After integration and transformation one has the following dependence

$$T_e = T_0 \left(1 + \frac{1}{1 + (2\omega\tau_{rel})^2} \left((1 + 2\omega\tau_{tr}\omega\tau_{rel}) \cos(2\omega t) \right) + (2\omega\tau_{rel} + \omega\tau_{tr}) \sin(2\omega t) \right) \tag{6}$$

where

$$T_0 = \frac{e^2 E_0{}^2 \tau_{tr} \cdot \tau_{rel}}{m \cdot 2} \cdot \frac{1}{\left(1 + (\omega\tau_{tr})^2\right)} \tag{7}$$

is the average electron temperature.
As is following from Eq.(6), for DC field (ω=0) the electron temperature and DC electric field are coupled by Eq.(8)

$$T_{DC} = \frac{e^2 E_{DC}{}^2 \tau_{tr} \cdot \tau_{rel}}{m} \tag{8}$$

If $T_{DC}=T_{cr}$, where T_{cr} is satisfying Eq.(9) at DC field

$$\nu_i(T_{cr}) - \nu_a(T_{cr}) = 0 \tag{9}$$

then Eq.(10) defines the critical value of DC electric field

$$E_{DCcr} = \sqrt{T_{cr} \cdot \frac{m}{e^2 \tau_{tr}}} \tag{10}$$

and Eq.(7) can be represented by Eq.(11)

$$\frac{T_0}{T_{cr}} = \frac{E_0^{\ 2}}{E_{DCcr}^{\ 2}} \cdot \frac{1}{2 \cdot \left(1 + (\omega \tau_{tr})^2\right)} \tag{11}$$

In literature is often used the ratio

$$\delta = \frac{\tau_{tr}}{\tau_{rel}}, \qquad \delta << 1$$

Let us designate

$$\eta = \omega \tau_{rel}, \tag{12}$$

and let us represent the Eq. (6) in more compact form

$$T_e(\eta, E_0, t) = T_{cr} \cdot \left(\frac{E_0}{E_{DCcr}}\right)^2 \cdot \frac{1}{2\left(1 + (\delta\eta)^2\right)} \cdot \left(1 + \frac{1}{1 + (2\eta)^2}\left(\left(1 + 2\delta\eta^2\right)\cos(2\omega t)\right) + (2 + \delta)\sin(2\omega t)\right) \tag{13}$$

If one knows the temporary electron temperature periodical evolution then he can determine averaged in time effective ionization frequency (so as difference of ionization and attachment frequencies) using known dependence of the summed ionization frequency dependence on temperature $\nu_{ia}(T_e)$

$$\nu_{ef}(E_0) = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \left(\nu_i\left(T_e(\eta, E_0, t)\right) - \nu_a\left(T_e(\eta, E_0, t)\right)\right) dt\,. \tag{15}$$

Equivalence to zero of averaged ionization frequency determines the MW critical breakdown field $\mathbf{E_{0cr}}$

$$\nu_{ef}(E_0) = 0\,. \tag{16}$$

The Eq.(16) was integrated numerically with use the following expressions for approximation of dependencies of ionization and dissociate attachment frequencies on electron temperature

$$\nu_i(T_e) = \nu_a(T_{cr}) \cdot \left(\frac{T_e}{T_{cr}}\right)^{\frac{5.3}{2}}, \qquad \nu_a(T_e) = \nu_a(T_{cr}) \cdot \left(\frac{T_e}{T_{cr}}\right) \tag{17}$$

where

$\boldsymbol{T_{cr}}$ – is the critical temperature for DC electric field.

In **Fig.1** one can see a result of the numerical integration of the equation (16).

One has the simple but important conclusion from obtained dependences.

At η>1 (i.e. at p<100 Torr for λ=8.9 cm) the breakdown threshold corresponds to equality of **effective value** of electric field E_{ef} and the critical value of the constant field E_{DCcr}

$$E_{ef} \equiv \frac{E_0}{\sqrt{2 \cdot \left(1 + (\omega \tau_{tr})^2\right)}} = E_{DCcr}, \qquad \omega \tau_{rel} >> 1 \tag{18}$$

At η<1 (i.e. at p>100 Torr for λ=8.9см) the breakdown threshold corresponds to equality of **amplitude value** of electric field E_0 and the critical value of the constant field

$$E_0 \approx E_{DCcr}, \qquad \omega\tau_{rel} << 1 \tag{19}$$

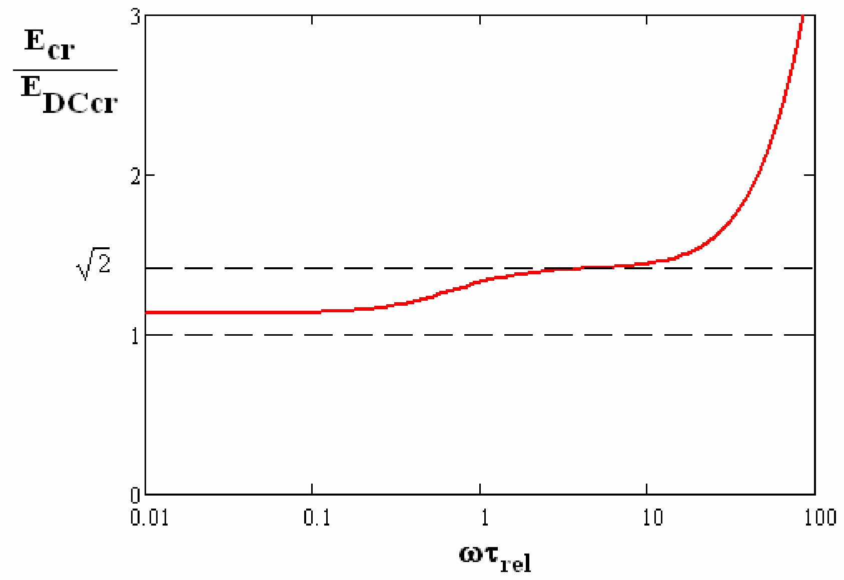


Figure 1. A result of numerical integration of the equation (16)

The obtained result is physically clear. The ionization frequency depends on electron temperature near its critical value very strongly. For positive balance of ionization the small excess of maximum value of electron temperature at critical value is enough. If the energy relaxation time is small comparative to MW field period, divided on 2π, then electron temperature is pulsating together with momentary value of Ohm losses. It means that maximum electron temperature achieves critical value if MW electric field amplitude equals to critical value for DC case. In opposite case, when energy relaxation time is large, pulsation of electron temperature is negligible and its average value must exceed the critical value. Consequently for breakdown the equality of effective value of MW field and DC breakdown value is being demanded.

III. The numerical calculation

The simplified theory is based on supposition, that parameters τ_{tr} and τ_{rel} are constants. At this condition Eq.(2) and Eq.(4) are linear and can be solved analytically. In reality these parameters are function of electron temperature so solution can be obtained only numerically.

For specification of result of simplified theory the numerical integration of the Eq.(2) and Eq.(4) have been undertaken on example of air. The real dependences of characteristic times on electron temperature

$$\tau_{tr} = \tau_{tr}(T_e) \tag{20}$$

and

$$\tau_{rel} = \tau_{rel}(T_e) \tag{21}$$

were calculated with a help of known cross sections of electron impact ionization, attachment, elastic scattering, molecular and electron excitation.

Used model of ionization balance in air allows defining of critical values of electron temperature

$$T_{e_{cr}} = 2.34, eV \tag{22}$$

the critical value of DC electric field

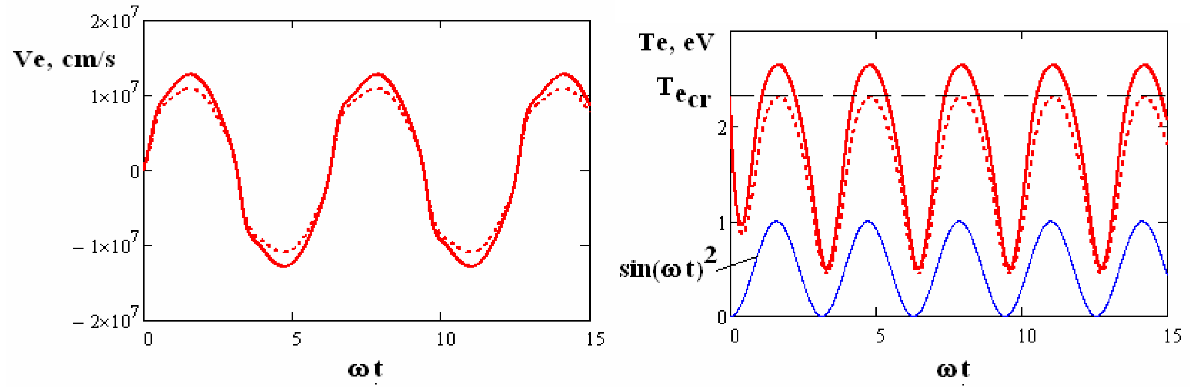
$$E_{DCcr} = 40 \cdot p \text{, V/cm,} \tag{23}$$

and value of parameter δ at critical electron temperature

$$\delta \approx 0.03\,. \tag{24}$$

Calculated magnitudes coincide with well known experimental data quite satisfactory[4]. It confirms the adequacy of designed model.

The modeling shows that at high pressure electron temperature is varying with very high amplitude. For example, Figure 2 demonstrates the temporary evolution of electron drift velocity and temperature at air pressure 760 Torr in MW electric field of critical value and λ=8.9cm.



(a) (b)

Figure 2. Temporary evolution of electron drift velocity (a) and temperature (b) at air pressure 760 Torr in MW electric field of critical value, λ=8.9cm. $E=E_{cr}$ - dashed line, $E=1.2E_{cr}$ – solid line.

In **Figure 3** the **r**esults of modeling with accounting of dependences Eq.(13) and Eq.(14), calculated for the wavelength 8.9cm and 2.5cm in diapason of pressure, are shown. The modeling results represented by points can be compared with data of simplified theory, represented by solid lines. Red and blue colors correspond to wavelength 8.9cm and 2.5cm. At that for calculation of characteristic times of scattering and energy relaxation for use in simplified theory were used formulas

$$\tau_{tr} \approx \frac{1.6\cdot 10^{-10}}{p}, s \tag{25}$$

$$\tau_{tr} \approx \frac{\tau_{tr}}{\delta}, \tag{26}$$

where p is air pressure, Torr.

One can see an excellence agreement of simplified theory with modeling data. With pressure increase the transition from effective to amplitude value in equality to critical constant field E_{DCcr} takes place. At pressure below 10 Torr the mode of rare collisions is realized.

Generally the dependence of critical value of amplitude of MW electric field is satisfactory approximated by formula Eq.(27) used together with Eq.(25) and Eq.(26)

$$E_{cr} = \sqrt{\frac{1.2 + 2(\omega\tau_{rel})^2}{1+(\omega\tau_{rel})^2}\left(1+(\omega\tau_{tr})^2\right)}, \tag{27}$$

In Fig.3 the dependences, calculated by approximation Eq.(27), are painted by dashed lines and almost fully coincide with solid lines. The formula Eq.(27) can be recommended for estimation of breakdown threshold in MW field.

Obtained result is physically quite transparent and does not require explanations. It was important to clarify at which gas pressure value takes place changing of the amplitude on effective value. Equation (27) gives possibility to know it.

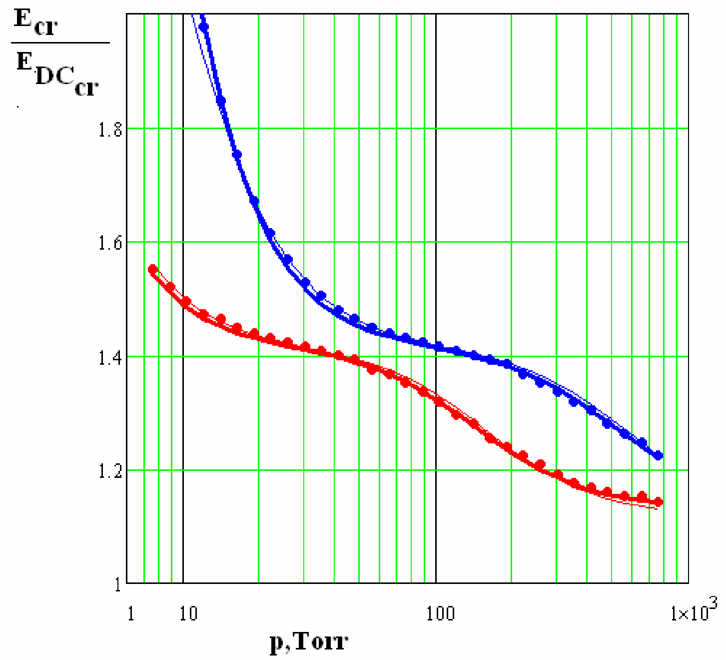


Figure 3. The rate of critical value of MW electric field amplitude to DC one in dependence on air pressure. Red color - λ=8.9cm, blue clor - λ=2.5cm. Modeling – points, simplified theory – solid lines.

IV. Discussion

The developed theory of breakdown threshold in MW radiation at middle and high gas pressure is interesting not only for more deeply understanding of breakdown physics but for various technology applications. The most reliable knowledge of exact value of breakdown threshold is topical for many different areas and technologies

For example, it is very important to know exactly the amplitude spatial distribution of MW radiation at study of overcritical and subcritical discharges. It is strongly nonlinear process so knowledge of absolute value of origin field amplitude is necessary. Measurement of field amplitude absolute value in a beam of MW radiation or in a complicated MW device is extremely difficult task. The most reliable method of amplitude absolute value measurement has been designed and successfully is used in MW discharge experiments. The essence of this method is following. In a point of measurement a small metal ball is being placed. The metal ball creates the local field amplitude increase on its surface in three times relative to value unperturbed by ball. Then gas pressure in test camera is decreasing up to breakdown value. The breakdown takes place on ball surface because field here is maximal. If the dependence of breakdown on gas pressure is known, the measured value of pressure is determining electric field amplitude. It is clear that accuracy of this measurement method is being defined by accuracy of the used dependence of breakdown amplitude on gas pressure.

Acknowledgments

The work is performed with financial support of EOARD (Projects ISTC # 3784p). Author sincerely thanks Dr. David M. Van Wie and Dr. Julian Tishkoff for displayed interest and Dr. Igor Esakov and Mr. Lev Grachev for fruitful discussions.

References

[1] S. Brown. *Elementary processes in gas discharge plasma*. Moscow. Atomizdat. 1961

[2] A.D.MacDonald. *Microwave Breakdown in Gases*. New York. 1965.

[3] Yu.P.Raizer. *Physics of gas discharge*. Moscow, “Nauka”, 1987.

[4] A.V.Eletsky, L.A.Galkina, B.M.Smirnov. *Transfer phenomena in weakly ionized gases*. Moscow. Atomizdat. 1975